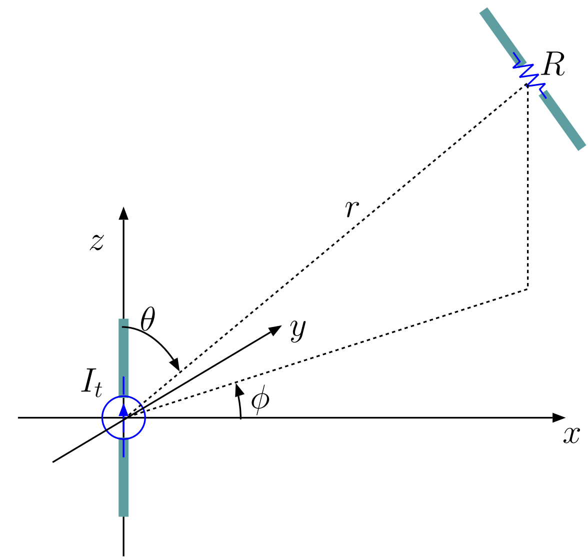
36 Antenna reception and links

Elementary description of antenna reception and links:



Current I_r that flows through the resistor R connected across the antenna terminals is induced by the tangential components of incident wave fields on the conducting arms of the dipole antenna; this response is a straightforward consequence of the need to maintain tangential boundary conditions on the antenna surface.

- Consider a pair of *identical* short dipole antennas in free space, one located at the origin $(x, y, z) = (0, 0, 0)$ and the other at a distance r away from the origin at zenith and azimuth angles of (θ, ϕ) as shown in the margin. Furthermore:
  - ant.1 at the origin, e.g. a $\hat{z}$-polarized short-dipole, sees ant.2 located at angles (θ, ϕ), but
  - ant.2's orientation is adjusted so that it always sees ant.1 at fixed angles of, say, $(\theta_2, \phi_2) = (90°, 0)$, defined in its own coordinate system, for all possible locations (r, θ, ϕ).

- First, ant.1 is driven with an input current (phasor) I_t and a time-average input power

$$P_t = \frac{1}{2}|I_t|^2 R_{rad},$$

while ant.2, terminated by a resistor R, puts a current I_r through R into which it delivers an average power

$$P_r = \frac{1}{2}|I_r|^2 R \equiv S_{inc} A(\theta_2, \phi_2)$$

where:

1.

$$S_{inc} = \frac{P_t}{4\pi r^2} G(\theta, \phi)$$

is the incident power density (the magnitude of the time-average Poynting vector) of the field arriving from ant.1 expressed in terms of the antenna gain $G(\theta, \phi)$ evaluated in the angular direction of ant.2, and

2. $A(\theta_2, \phi_2)$ is called the **antenna effective area** and is *defined to be* the conversion factor between the **received power** P_r (W) and the **incident power density** S_{inc} (W/m^2) .

Our aim is to identify the effective area function $A(\theta, \phi)$ in terms of the antenna gain function $G(\theta, \phi)$ and two constraints, one regarding R, and the other regarding the antenna polarization.

Once that is accomplished, the receiving properties of antennas will be relatively easy to understand.

- Combining the expressions above we note that

$$P_r = \frac{P_t}{4\pi r^2} G(\theta, \phi) A(\theta_2, \phi_2).$$

- Now, swap the source I_t and the termination resistance R between the two (identical) sets of antenna terminals, so that now ant.2 becomes the "transmitter" and ant.1 is the "receiver".

Power received by ant.1 in response to S_{inc} from ant.2 in that case can be expressed as

$$P_r = \frac{P_t}{4\pi r^2} G(\theta_2, \phi_2) A(\theta, \phi)$$

by using similar arguments — in this expression $G(\theta_2, \phi_2)$ is the antenna gain in the direction of ant.1 as seen by ant.2, whereas $A(\theta, \phi)$ is the antenna effective area for the direction ant.2 appears with respect to ant.1.

Since the two cases considered above are *in essence* identical — in each case the receiving antenna is exposed to the same tangential field arriving from the transmitting antenna — the very same power P_r must be received by R in each case; hence, it is necessary that

$$G(\theta, \phi) A(\theta_2, \phi_2) = G(\theta_2, \phi_2) A(\theta, \phi),$$

for all possible (θ, ϕ), implying that

$$\frac{A(\theta, \phi)}{G(\theta, \phi)} = \frac{A(\theta_2, \phi_2)}{G(\theta_2, \phi_2)} = \text{const.}$$

Therefore, we conclude that for short dipole antennas, the effective area function must be given as

$$A(\theta, \phi) = K_a G(\theta, \phi),$$

where K_a is a *scaling constant* independent of (θ, ϕ) *that remains to be determined.*

- **Later we will show that the required scaling is**

$$K_a = \frac{\lambda^2}{4\pi} \quad \Rightarrow \quad A(\theta,\phi) = \frac{\lambda^2}{4\pi} G(\theta,\phi)$$

not only for short-dipoles, but for all types of antennas[1].

That is a crucial finding which implies that the **reciprocal** relation of P_r to P_t found above **between** the cases when the transmitting and receiving roles of identical short-dipoles are swapped, will also be valid even when the antennas are non-identical (e.g., antenna 1 a short dipole, antenna 2 a broadside 1D array of half-wave dipoles).

We can then succinctly express the reciprocal relation in the form

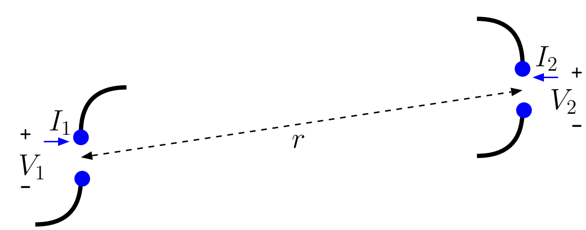
$$P_r = P_t \frac{\lambda^2 G_t G_r}{(4\pi r)^2}, \quad \text{(Friis transmission formula)}$$

where G_t and G_r refer to the gain of the (arbitrary) antennas used for transmission and reception in the directions of direct contact between the antennas in any communications link, whereas P_r and P_t are, respectively, the average power transmitted and the average available power of the receiving antenna.

  – Reciprocity is a convenient property of antenna behaviour because it allows for one of the link antennas to be a low-gain antenna at

[1]However, this result for K_a requires the termination R to be a "matched load" to the antenna and also effective area $A(\theta,\phi)$ be defined for the reception of the "co-polarized" component of the incident field. The concepts of a ***matched load*** and a ***co-polarized field*** will be explained in detail later on.

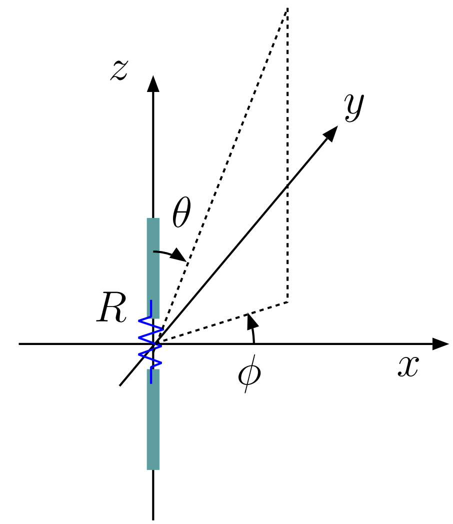
$$A(\theta,\phi) = \frac{\lambda^2}{4\pi} G(\theta,\phi)$$



Friis formula and transmission gain

$$\frac{P_r}{P_t} = \frac{\lambda^2 G_t G_r}{(4\pi r)^2}$$

the expense of the gain of the second antenna, for a fixed value of overall **transmission gain** P_r/P_t.



Example 1:

1. Consider a $\hat{z}$-polarized short-dipole antenna at the origin terminated by a load $Z_L = Z_a^*$, where Z_a is the input impedance of the same dipole antenna when it is used for transmission purposes.

   Because $Z_L = Z_a^*$ (and not an arbitrary R — see margin) we will be able to compute the time-averaged power P_r delivered by the antenna into the load Z_L using the simple procedure explained at the end of this example (based on the ideas already developed above).

2. Also consider the same $\hat{z}$-polarized short-dipole antenna responding to incident plane TEM waves which have electric field vectors polarized in $\hat{\theta}$ direction.

   Because the polarization direction $\hat{\theta}$ of the incident electric field is co-polarized with the receiving antenna, we will be able to compute P_r using the simple procedure explained next.

To compute the time-averaged power P_r delivered by the $\hat{z}$-polarized antenna to its matched termination $Z_L = Z_a^*$ it is sufficient to multiply

$$A(\theta, \phi) = \frac{\lambda^2}{4\pi} G(\theta, \phi)$$

with

$$S_{inc} = \frac{|\tilde{\mathbf{E}}_{inc}|^2}{2\eta};$$

that is

$$P_r = S_{inc} A(\theta, \phi).$$

Had it been the case that $Z_L \neq Z_a^*$ or the polarization of the electric field not $\hat{\theta}$, the formula for P_r would have been different.

Example 2: This is one possible receiving scenario compatible with what was described in Example 1.

The incident TEM wave field is

$$\tilde{\mathbf{E}}_{inc} = 120\pi \frac{\hat{z} - \hat{x}}{\sqrt{2}} e^{j2\pi(x+z)} \,\mathrm{V/m}.$$

This field has a wave vector

$$\mathbf{k} = -2\pi(\hat{x} + \hat{z}) \,\mathrm{rad/m},$$

indicating the antenna sees the incident wave coming from direction $\theta = 45^\circ$ with a polarization direction $\hat{\theta}$. Since

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{|\mathbf{k}|} = \frac{2\pi}{2\pi\sqrt{2}} = \frac{1}{\sqrt{2}} \,\mathrm{m}$$

and

$$A(\theta, \phi) = \frac{\lambda^2}{4\pi} G(45^\circ, \phi) = \frac{1/2}{4\pi} \underbrace{(\frac{3}{2}\sin^2 45^\circ)}_{\textbf{short-dipole gain at } 45^\circ} = \frac{1.5}{16\pi} \,\mathbf{m}^2,$$

and, furthermore,

$$S_{inc} = \frac{|\tilde{\mathbf{E}}_{inc}|^2}{2\eta} = \frac{(120\pi)^2}{2 \times 120\pi} = 60\pi \,\mathrm{W/m}^2,$$

it follows that

$$P_r = S_{inc} A(\theta, \phi) = 60\pi \times \frac{1.5}{16\pi} = \frac{90}{16}\,\text{W}.$$

Assuming that the antenna is terminated with a matched load, it will deliver a time-averaged power of 90/16 W to its load.

Example 3: Another scenario compatible with what was described in Example 1.

The incident TEM wave field is

$$\tilde{\mathbf{E}}_{inc} = 120\pi \hat{z} e^{j2\pi x}\,\text{V/m}.$$

This field has a wave vector

$$\mathbf{k} = -2\pi \hat{x}\,\text{rad/m},$$

indicating the antenna sees the incident wave coming from direction $\theta = 90°$ with a polarization direction $\hat{\theta}$. Since

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{|\mathbf{k}|} = \frac{2\pi}{2\pi} = 1\,\text{m}$$

and

$$A(\theta, \phi) = \frac{\lambda^2}{4\pi} G(90°, \phi) = \frac{1}{4\pi} \underbrace{(\frac{3}{2}\sin^2 90°)}_{\textbf{short-dipole gain at } 90°} = \frac{3}{8\pi}\,\mathbf{m}^2,$$

and, furthermore,

$$S_{inc} = \frac{|\tilde{\mathbf{E}}_{inc}|^2}{2\eta} = \frac{(120\pi)^2}{2 \times 120\pi} = 60\pi\,\text{W/m}^2,$$

it follows that

$$P_r = S_{inc}A(\theta,\phi) = 60\pi \times \frac{3}{8\pi} = \frac{90}{4}\,\text{W}.$$

Assuming that the antenna is terminated with a matched load, it will deliver a time-averaged power of 22.5 W to its load.

Example 4: This scenario is incompatible with what was described in Example 1.

The incident TEM wave field is

$$\tilde{\mathbf{E}}_{inc} = 120\pi\hat{y}e^{j2\pi x}\,\text{V/m}.$$

This field has a wave vector

$$\mathbf{k} = -2\pi\hat{x}\,\text{rad/m},$$

indicating the antenna sees the incident wave coming from direction $\theta = 90°$. But the polarization direction of the wave is not $\hat{\theta}$, it is $\hat{\phi}$. Therefore, even though we have

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{|\mathbf{k}|} = \frac{2\pi}{2\pi} = 1\,\text{m}$$

and

$$A(\theta,\phi) \;=\; \frac{\lambda^2}{4\pi}G(90°,\phi) = \frac{1}{4\pi}\underbrace{(\frac{3}{2}\sin^2 90°)}_{\textbf{short-dipole gain at } 90°} = \frac{3}{8\pi}\,\mathbf{m}^2,$$

we need to conclude that

$$P_r = S_{inc}A(\theta,\phi) = 0$$

because S_{inc} that needs to be associated with a $\hat{\theta}$-polarized field is zero!

- **In Example 4 the incident field is *cross-polarized* with the receiving antenna and therefore there is no power transfer to the matched termination of the antenna.**

- **By contrast, in Examples 2 and 3 the incident fields were *co-polarized* with the receiving antenna and therefore average power calculation to the matched antenna termination was accomplished by following the recipe given in Example 1.**

Try understanding the meaning of co- and cross-polarized incident fields.

An incident field is said to be *co-polarized* when its electric field vector is aligned with the field the receiving antenna would radiate if it were being used in transmission mode !!!

- **In the upcoming lectures we will verify the important ideas introduced in this lecture — namely**

1. The *matched impedance* concept,
2. *Co-* and *cross-polarized* signals,
3. The *effective area* $A(\theta, \phi)$ and the relation
$$A(\theta, \phi) = \frac{\lambda^2}{4\pi} G(\theta, \phi),$$
4. *Friis transmission formula*
$$P_r = P_t \frac{\lambda^2 G_t G_r}{(4\pi r)^2} = P_t \frac{A_t A_r}{(\lambda r)^2}.$$