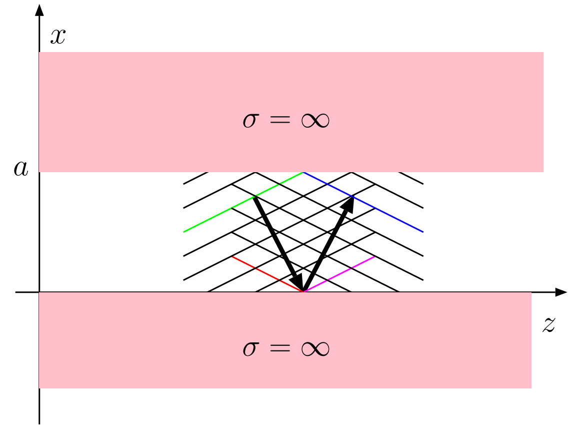
27 Parallel-plate waveguides: example problems



Summarizing the properties of guided modes of propagation in parallel-plate waveguides:

- TE_m and TE_m modes with the transverse field phasors

  $$\tilde{\mathbf{E}} = 2j\hat{y}E_o e^{-jk_z z}\sin(k_x x) \;\text{ and }\; \tilde{\mathbf{H}} = 2\hat{y}H_o e^{-jk_z z}\cos(k_x x),$$

  respectively, where

  $$k_x = \frac{m\pi}{a} \;\text{ and }\; k_z = \sqrt{k^2 - k_x^2} = \frac{\omega}{c}\sqrt{1 - \frac{f_c^2}{f^2}} = \frac{\omega}{c}\sqrt{1 - \frac{\lambda^2}{\lambda_c^2}},$$

  with **cutoff frequencies** and **wavelengths**

  $$f_c = \frac{mc}{2a} \;\text{ and }\; \lambda_c = \frac{2a}{m},$$

  respectively, satisfy the zero tangential $\tilde{\mathbf{E}}$ boundary conditions on $x = 0$ and $x = a$ plates of the guide.

  - TE_0 mode does not exist but TM_0=TEM does and it is dispersionless.
  - All TE_m and TM_m modes are dispersive for $m \geq 1$, and propagate only if $f > f_c$, or, equivalently, $\lambda < \lambda_c$.
  - Non-propagating modes are evanescent and have an attenuation constant $|k_z|$.

- Also TE$_m$ and TE$_m$ mode fields have guide impedances

$$\eta_{TE} = \frac{E_y}{-H_x} = \frac{\eta_o}{\sqrt{1-\frac{f_c^2}{f^2}}} \text{ and } \eta_{TM} = \frac{E_x}{H_y} = \eta_o\sqrt{1-\frac{f_c^2}{f^2}}$$

  relating the transverse field components of the guided modes.

All the results summarized above are for air-filled waveguides, but they can be readily modified, by replacing c and η_o with c/n and η, respectively, in the case of dielectric-filled waveguides.

Example 1: Consider a dielectric-filled parallel-plate waveguide with $a = 2$ cm. The permeability of the dielectric filling is μ_o and its refractive index is $n = 1.5$.

1. Which TE$_m$ and TM$_m$ modes can propagate a 12 GHz signal in the waveguide?
2. What would be the associated cutoff wavelengths in each case?
3. What would be the associated group velocities in each case? — here assume a modulated 12 GHz carrier with a narrow modulation bandwidth.

Solution: Unguided propagation velocity for the dielectric filling the waveguide is

$$v = \frac{c}{n} = \frac{3\times10^8\,\text{m/s}}{1.5} = 2\times10^8\,\frac{\text{m}}{\text{s}}.$$

Using v in place of c in the cutoff frequency formula for TE$_m$ and TM$_m$ modes we find

$$f_c = \frac{mv}{2a} = \frac{m\times2\times10^{10}\,\text{cm/s}}{2\times2\,\text{cm}} = m5\times10^9\,\text{Hz} = 5m\,\text{GHz}.$$

1. $f = 12$ GHz exceeds the cutoff frequencies of TE_m and TM_m for $m = 1$ and 2, but not 3. Therefore, the propagating (i.e., non-evanescent) modes at $f = 12$ GHz are TM_0, TE_1, TM_1, TE_2, and TM_2.

2. Cuttof-wavelength are given by the equation

$$\lambda_c = \frac{2a}{m}$$

and do not depend on the dielectric filling. They are, with $a = 2$ cm,

$$\lambda_c = 4\,\text{cm for TE}_1\text{=TM}_1 \text{ and } \lambda_c = 2\,\text{cm for TE}_2\text{=TM}_2.$$

The cutoff wavelength is ∞ for TEM mode (which does not have a cutoff condition).

3. Group velocities are given by the equation

$$v_g = v\sqrt{1 - \frac{f_c^2}{f^2}}$$

where $v = c/n$. For the non-dispersive TEM mode with $f_c = 0$ the group velocity is $v_g = 2 \times 10^8$ m/s. For TE_1 and TM_1 modes

$$v_g = v\sqrt{1 - \frac{5^2}{12^2}} = 1.82 \times 10^8\,\text{m/s}.$$

For TE_2 and TM_2 modes

$$v_g = v\sqrt{1 - \frac{10^2}{12^2}} = 1.11 \times 10^8\,\text{m/s}.$$

Example 2: Consider an air-filled parallel-plate waveguide with $a = 3$ cm. Calculate the guide wavelength λ_g or the attenuation rate in dB/cm of the TE_1 mode in the guide — whichever appropriate — if the operating wavelength of the mode is (a) $\lambda = 3$ cm, and (b) $\lambda = 12$ cm.

Solution: The cutoff wavelength of TE_1 mode in the guide is

$$\lambda_c = \frac{2a}{m} = \frac{2 \times 3\,\mathrm{cm}}{1} = 6\,\mathrm{cm}.$$

(a) For $\lambda = 3$ cm, $\lambda < \lambda_c$, and, therefore, the TE_1 mode is propagating. The propagation constant, that is k_z, is

$$k_z = k\sqrt{1 - \frac{\lambda^2}{\lambda_c^2}} = \frac{2\pi}{\lambda}\sqrt{1 - \frac{\lambda^2}{\lambda_c^2}}$$

and the guide wavelength is

$$\lambda_g = \frac{2\pi}{k_z} = \frac{\lambda}{\sqrt{1 - \frac{\lambda^2}{\lambda_c^2}}} = \frac{3\,\mathrm{cm}}{\sqrt{1 - \frac{3^2}{6^2}}} = \frac{3\,\mathrm{cm}}{\sqrt{1 - \frac{1}{4}}} = \frac{3\,\mathrm{cm}}{\sqrt{\frac{3}{4}}} = 2\sqrt{3}\,\mathrm{cm}.$$

(b) For $\lambda = 12$ cm, $\lambda > \lambda_c$, and, therefore, the TE_1 mode is evanescent. The attenuation constant is $|k_z|$, where

$$k_z = k\sqrt{1 - \frac{\lambda^2}{\lambda_c^2}} = \frac{2\pi}{\lambda}\sqrt{1 - \frac{\lambda^2}{\lambda_c^2}} = \frac{2\pi}{12}\sqrt{1 - \frac{12^2}{6^2}} = \frac{\pi}{6}\sqrt{-3} = \pm j\frac{\pi}{2\sqrt{3}}\,\mathrm{rad/cm}.$$

Therefore, the attenuation rate is

$$20\log_{10} e^{|k_z|} = |k_z| 20\log_{10} e = \frac{\pi}{2\sqrt{3}} \times 8.686 \approx 7.88\,\mathrm{dB/cm}.$$

Example 3: A parallel-plate waveguide with $a = 3$ cm is air filled for $z < 0$ but it is filled with a dielectric for $z > 0$ which has $\mu = \mu_o$ and a refractive index $n = 1.5$. If a TE_1 mode wave field with $\lambda = 3$ cm is incident from the air-filled region on the interface at $z = 0$, what fraction of the time-averaged incident power will be transmitted into the $z > 0$ region of the guide?

Solution: The cutoff wavelength of TE_1 mode in the guide is

$$\lambda_c = \frac{2a}{m} = \frac{2 \times 3\,\text{cm}}{1} = 6\,\text{cm}.$$

For $\lambda = 3$ cm, the intrinsic impedance of the TE_1 mode fields is therefore

$$\eta_{TE1} = \frac{\eta_o}{\sqrt{1 - \frac{\lambda^2}{\lambda_c^2}}} = \frac{120\pi}{\sqrt{1 - \frac{3^2}{6^2}}} = \frac{120\pi}{\sqrt{\frac{3}{4}}} = \frac{240\pi}{\sqrt{3}}\,\Omega$$

in the air filled section.

Within the dielectric region the operation wavelength is $\lambda_2 = \lambda/n = 3/1.5 = 2$ cm, and, therefore the intrinsic impedance is

$$\eta_{TE2} = \frac{\eta_o/n}{\sqrt{1 - \frac{\lambda_2^2}{\lambda_c^2}}} = \frac{120\pi/1.5}{\sqrt{1 - \frac{2^2}{6^2}}} = \frac{80\pi}{\sqrt{\frac{8}{9}}} = \frac{240\pi}{\sqrt{8}}\,\Omega.$$

Thus, using a transmission line analogy, the reflection coefficient at the interface is

$$\Gamma = \frac{\eta_{TE2} - \eta_{TE1}}{\eta_{TE2} + \eta_{TE1}} = \frac{\frac{1}{\sqrt{8}} - \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{8}} + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - \sqrt{8}}{\sqrt{3} + \sqrt{8}} = -0.24,$$

which is the transverse electric field amplitude of the reflected wave in the air filled region divided by the incident electric field amplitude.

Consequently, the fraction of the incident time-averaged power reflected back from the interface is the reflectance

$$|\Gamma|^2 \approx 0.058,$$

and

$$1 - |\Gamma|^2 \approx 0.942$$

represents the transmittance, the fraction of the incident time-averaged power transmitted into the dielectric filled region.