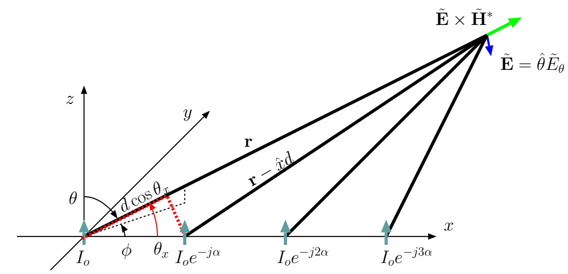
13 Arrays and feed networks

Performance of antenna arrays depends on our ability to feed the array elements with input currents having accurate phase relationships. This can be accomplished by using appropriately designed "feed networks" consisting of transmission line (TL) segments. We continue our study of antenna arrays with examples illustrating feed network design issues.



Example 1: Consider a 4-element phased array with $\hat{z}$-polarized short dipole elements, progressive phasing with increments α, and element-to-element spacings $d = \frac{\lambda}{2}$ along the x-axis. It is desired that the array has a gain maximum on $\theta = 90°$ plane along $\phi = 45°$ directions. Determine α such that $I_1 = I_0 e^{-j\alpha}$ and suggest a TL feed network that can be used to distribute the required input currents of the array elements.

Solution: Let $\tilde{\mathbf{E}}_0(\mathbf{r})$ denote the far-field phasor due to the array element at the origin. The phasor due to the element at $(d, 0, 0)$ can then be expressed (in the far-field) as

$$\tilde{\mathbf{E}}_1(\mathbf{r}) = \tilde{\mathbf{E}}_0(\mathbf{r})e^{-j\alpha}e^{jkd\cos\theta_x} = \tilde{\mathbf{E}}_0(\mathbf{r})e^{-j\alpha}e^{jkd\sin\theta\cos\phi}.$$

Since we are interested in $\theta = 90°$ case, we can simplify this as

$$\tilde{\mathbf{E}}_1(\mathbf{r}) = \tilde{\mathbf{E}}_0(\mathbf{r})e^{j(kd\cos\phi - \alpha)}.$$

Since we want constructive interference in $\phi = 45°$ direction, we will demand that

$$e^{j(kd\cos\phi - \alpha)} = 1 \quad \Rightarrow \quad \alpha = kd\cos\phi \;\text{ for }\; \phi = 45°,$$

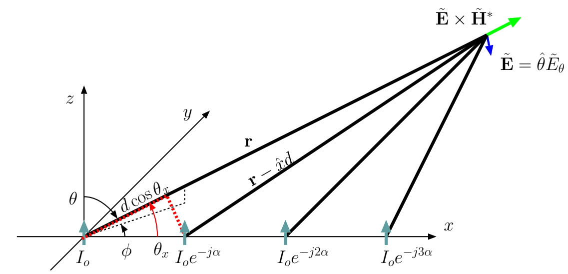
i.e,

$$\alpha = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos 45° = \frac{\pi}{\sqrt{2}}\,\text{rad}.$$

Thus, the required current inputs of the array elements are

$$I_n = I_0 e^{-jn\frac{\pi}{\sqrt{2}}}, \quad n = 0, 1, 2, 3.$$

With this phasing, radiation coming from all four elements will interfere constructively along the $\phi = 45^\circ$ direction.



Designing the feed network: The required phase delays

$$\alpha = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos 45^\circ = \frac{\pi}{\sqrt{2}}\,\text{rad}$$

to be applied progressively are *in effect* to compensate for the fact that propagation distance from **element 1** to the observation point (along the $\phi = 45^\circ$ line) is shorter than the distance from **element 0** by an amount

$$\Delta = d\cos\phi = \frac{\lambda}{2}\cos 45^\circ.$$

Let's make the current signal arriving at the input terminals of **element 1** travel on a TL (with propagation speed $v = c$) an *extra distance* Δ compared to the current going to **element 0** (coming from the same source) — that procedure, repeated progressively for all the elements, will produce the required current inputs

$$I_n = I_0 e^{-jn\frac{\pi}{\sqrt{2}}}, \quad n = 0, 1, 2, 3,$$

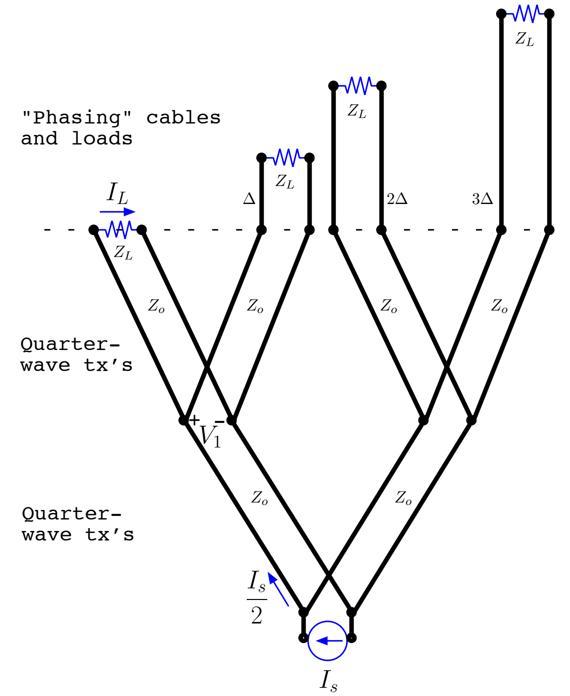
derived from a common current source, provided reflected waves from the elements can be avoided.

To avoid reflections from the antenna elements it is necessary to match the antenna impedance Z_L to the characteristic impedance of the TL (e.g., single-stub tuning).

Assume that all the elements have been identically matched to a TL with a characteristic impedance Z_o. Then we can connect the elements to a common current source I_s via a corporate-ladder network shown in the margin and have

$$I_n = \underbrace{-\frac{I_s}{2}}_{I_0} e^{-jn\frac{\pi}{\sqrt{2}}}, \quad n = 0, 1, 2, 3.$$

The verification of this formula would require the use of *quarter-wave transformation formulae* for impedance and terminal voltage and currents reviewed next.



- As shown in the margin a quarter-wave transformer with a characteristic impedance Z_o transforms a load impedance Z_L into an input impedance

$$Z_{in} = \frac{Z_o^2}{Z_L}.$$

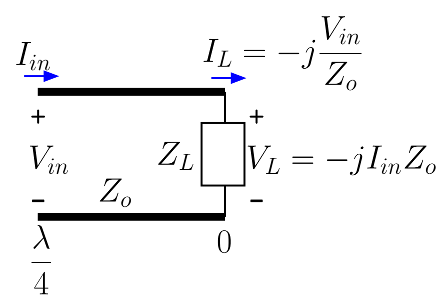
Also, an input voltage V_{in} is transformed to a load current

$$I_L = -j\frac{V_{in}}{Z_o}$$

independent of load impedance Z_L, while an input current I_{in} is transformed into a load voltage

$$V_L = -jZ_oI_{in}.$$

Quarter-wave transformer:



$$Z_{in}Z_L = Z_o^2$$

- Assuming $Z_L = Z_o$ and applying the quarter-wave transformer formulae repeatedly we note that

1. Junction voltage

$$V_1 = -j\frac{I_s}{2}Z_o;$$

2. Element 1 current

$$I_L = -j\frac{V_1}{Z_o} = -j\frac{(-j\frac{I_s}{2}Z_o)}{Z_o} = -\frac{I_s}{2}.$$

3. Element n current is then

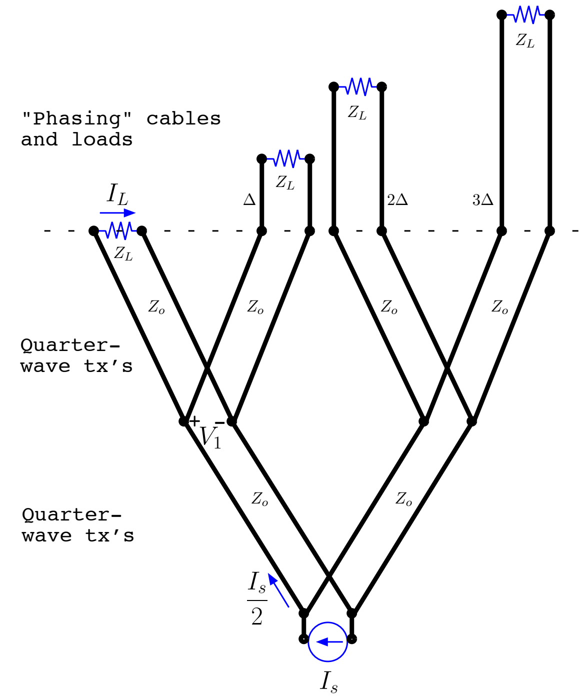
$$I_n = I_L e^{-jnk\Delta} = I_L e^{-jn\frac{\pi}{\sqrt{2}}}$$

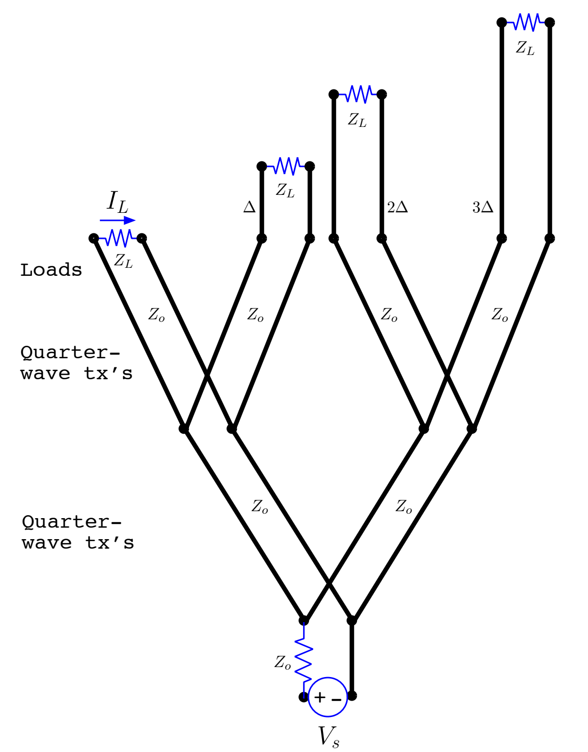
as required.

Question: Why did we have to assume $Z_L = Z_o$ above?

Answer: Because otherwise the impedance of the two branches seen by I_s may have been different, and we would not necessarily have equal currents $\frac{I_s}{2}$ flowing into the two branches of the circuit.

Exercise: In the diagram on the margin we have a voltage source V_s in series with a source impedance Z_o feeding an array feed network. Determine I_n using quarter-wave transformation rules assuming $Z_L = Z_o$.





Spatial Fourier transforms of current distributions:

- Consider an array of identical dipoles positioned along the x-axis at locations $x_n = nd$ having input currents I_n, with n in the interval $0, 1, \cdots N-1$.

- The far-field electric field of the array (in paraxial approximation) is then

$$\begin{aligned}\tilde{\mathbf{E}}(\mathbf{r}) &= \tilde{\mathbf{E}}_0(\mathbf{r})[1 + \frac{I_1}{I_0}e^{jkd\cos\theta_x} + \frac{I_2}{I_0}e^{j2kd\cos\theta_x} + \cdots + \frac{I_{N-1}}{I_0}e^{j(N-1)kd\cos\theta_x}] \\ &= \tilde{\mathbf{E}}_0(\mathbf{r})\sum_{n=0}^{N-1}\frac{I_n}{I_0}e^{jnkd\cos\theta_x}\end{aligned}$$

  so that the array factor is

$$\text{A.F.} = \sum_{n=0}^{N-1}\frac{I_n}{I_0}e^{jk\cos\theta_x nd},$$

  a discrete Fourier transform of the sequence $\frac{I_n}{I_0}$ representing the "illumination pattern" of the array into a spatial-frequency domain of $k_x \equiv k\cos\theta_x$.

- Compare the A.F. with the effective length of a single array element (from Lecture 8)

$$\ell = \int \frac{\tilde{I}(z)}{I_o}e^{jk\cos\theta z}dz,$$

  which is also a spatial Fourier transform into the domain $k_z \equiv k\cos\theta$.

- Therefore the system gain that consists of the products of the A.F. and element ℓ is in effect a spatial Fourier transform of the "current distributions" or so-called "illumination pattern" on antenna surfaces of the system.
  - This link will be examined more closely in antenna and imaging courses starting with ECE 454.