$\mu = 61.125 \quad \sigma = 12.97 \quad med = 63$

ECE 350	Fields and Waves II	Fall 2019
University of Illinois		Kudeki

Exam 3

Friday, Nov 22, 2019 — Noon-12:55 PM

Name:	Solution
Section:	12 Noon

Please clearly PRINT your name in CAPITAL LETTERS and circle your section in the above boxes.

This is a closed book exam. You are allowed to bring three sheets of notes and a calculator — both sides of the sheet may be used. Please show all your work and make sure to include your reasoning for each answer. All answers should include units wherever appropriate.

Problem 1 (25 points)	
Problem 2 (25 points)	
Problem 3 (25 points)	
TOTAL (75 points)	

1. **This problem has three independent parts:**

a) A narrowband pulsed carrier is propagating in a collisionless plasma. The carrier frequency is 10 MHz and the *phase* and *group* velocities for the pulsed carrier are known to be $v_p = \frac{5}{3}c$ and $v_g = \frac{3}{5}c$, respectively, where c is the speed of light in free space. Determine:

i. (4 pts) The wavelength of the 10 MHz carrier.

$f = 10$ MHz, $V_p = \frac{5}{3}c$, $V_g = \frac{3}{5}c$ $\quad n = \frac{3}{5}$

$\lambda f = V_p = \frac{5}{3}c \Rightarrow \lambda = \frac{\frac{5}{3} \times 3\times10^8 \text{ m/s}}{10^7\ 1/s} = 50$ m

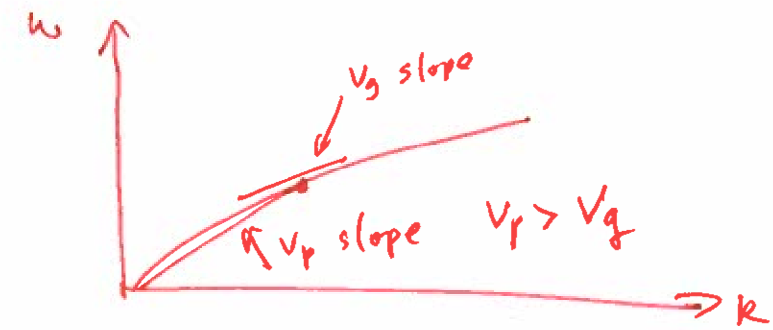
ii. (4 pts) Transit time of the pulse across a distance of 300 km in the medium.

$T = \frac{300 \text{ km}}{V_g} = \frac{300\times10^3 \text{ m}}{\frac{3}{5}\, 300\times10^6 \frac{m}{s}} = \frac{5}{3}\times10^{-3}\, s = \frac{5}{3}$ ms

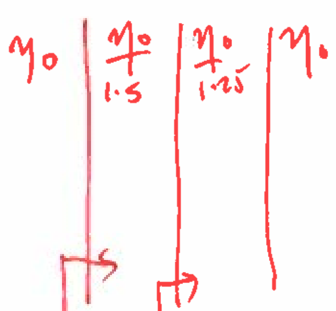
iii. (4 pts) The plasma frequency f_p in the plasma in MHz units.

$n = \sqrt{1 - \frac{f_p^2}{f^2}} = \frac{3}{5} \Rightarrow 1 - \frac{f_p^2}{f^2} = \frac{9}{25} \Rightarrow \frac{f_p^2}{f^2} = \frac{16}{25} \Rightarrow f_p = f\,\frac{4}{5} = 8$ MHz (10 MHz)

b) (5 pts) Sketch a possible dispersion curve ω versus k for a medium where $v_p > v_g \neq 0$ over all $k \geq 0$. Briefly discuss your choice.



c) (8 pts) Two distinct dielectric plates of $\frac{\lambda}{4}$ thickness each and having the refractive index of $n = 1.5$ and 1.25, respectively, are pasted back to back. Calculate the numerical value of *reflectance* of the two-plate structure when it is embedded in vacuum.



$Z_{in}' = \eta_0\left(\frac{1.25}{1.5}\right)^2 = \eta_0\left(\frac{5}{6}\right)^2 = \eta_0\frac{25}{36}$

$\Gamma = \frac{\eta_0\frac{25}{36} - \eta_0}{\ldots + \ldots} = \frac{25-36}{25+36} = \frac{-11}{61} \simeq -0.1803$

$|\Gamma|^2 = 0.0325 = 3.25\%$

$Z_{in} = \frac{(\eta_0/1.25)^2}{\eta_0} = \frac{\eta_0}{1.25^2}$

$Z_{in}' = \frac{(\eta_0/1.5)^2}{\eta_0/1.25^2}$

$\lambda = 4\text{cm}$, Propagation if $\lambda < \lambda_c$

2. **This problem has two independent parts:**

a) Consider an **air-filled parallel-plate waveguide** with plate separation of $a = 5$ cm.

i. (5 pts) Which TE_m and TM_m modes can **propagate** (instead of being cut-off) in the guide for a signal with a $\lambda = 4$ cm carrier wavelength within the guide. Explain your solution.

$$\lambda_c = \frac{2a}{m} = \frac{10\text{cm}}{m} \Rightarrow \lambda_0 = \infty,\ \lambda_1 = 10\text{cm},\ \lambda_2 = 5\text{cm},\ \lambda_3 = 3.3\text{cm}$$

propagating: TM0, TM1, TE1, TM2, TE2

ii. (5 pts) If the waveguide if next filled with a dielectric with $\mu = \mu_o$ and $n = 2$, how will the carrier wavelength of the signal (at the original carrier frequency) in (i) change? What will be the new λ?

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{k_o n} = \frac{4\text{cm}}{n} \Rightarrow \lambda: 4\text{cm} \Rightarrow 2\text{cm}$$

iii. (5 pts) After the change in (ii) which additional TE_m and TM_m modes are found to be propagating in the guide.

... $\lambda_3 = 3.3$ cm, $\lambda_4 = 2.5$ cm, $\lambda_5 = 2$ cm.

additional propagation in TM3, TE3, TM4, TE4 ✓

TM5 & TE5 are at cutoff!! ← Not propagating energy/info.

b) (10 pts) An **air filled rectangular cavity** of dimensions $a = 2b = 4d = 4$ cm. Identify the three lowest resonance frequencies f_{mnl} of the cavity along with the mode designations associated with them, e.g., TE_{101}, TM_{110}, etc.

$a = 4$ cm, $b = 2$ cm, $d = 1$ cm $\Rightarrow f_{mnl} = \frac{c}{2}\sqrt{\left(\frac{m}{4}\right)^2 + \left(\frac{n}{2}\right)^2 + \left(\frac{l}{1}\right)^2}$

$c = 3\times10^{10}\ \frac{\text{cm}}{\text{s}}$: $= 15\text{ GHz}\sqrt{\left(\frac{m}{4}\right)^2 + \left(\frac{n}{2}\right)^2 + \left(\frac{l}{1}\right)^2}$

for small f_{mnl} keep l small !!!

✓ $f_{110} = 15\text{GHz}\sqrt{\frac{1}{16} + \frac{1}{4}} = 8.38\text{ GHz}$ ← TM_{110}

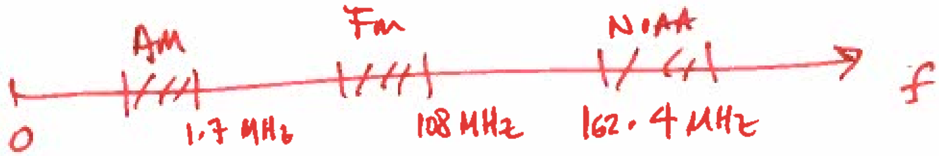
✗ $f_{011} = 15\text{GHz}\sqrt{\frac{1}{4} + 1} > 15\text{GHz}$

✗ f_{101} ---- $> 15\text{ GHz}$

✓ $f_{210} = 15\text{ GHz}\sqrt{\frac{1}{4} + \frac{1}{4}} = 10.66\text{ GHz}$ ← TM_{210}

✓ $f_{310} = 15\text{ GHz}\sqrt{\frac{9}{16} + \frac{1}{4}} = 13.52\text{GHz}$ ← TM_{310}

✓



3. Jane the cave climber, descending into a tight and deep cave system, is carrying a consumer radio that can receive the following frequency bands:

- AM broadcast band: 530 kHz – 1.7 MHz
- FM broadcast band: 88.0 MHz – 108.0 MHz
- NOAA weather radio: 162.4 – 165.55 MHz

After descending a substantial distance into the cave Jane notices that she can only receive a strong signal throughout the NOAA band — the entire AM and FM bands seem to be in evanescence. Modeling the cave system as an **air-filled metallic waveguide** with dimensions $a = b$, answer the following questions:

a) (5 pts) In which band, AM or FM, do you expect the reception to deteriorate more rapidly as the spelunker enters and proceeds in her descent within the cave? Explain your reasoning.

AM deteriorates fastest.
AM modes will have $\lambda \gg \lambda_c$ & in evanescence.

b) (8 pts) What is the minimum possible value of dimension a of the equivalent waveguide?

Lowest cutoff freq. → $f_{10} = \frac{mc}{2a}\Big|_{m=1} = \frac{c}{2a}$

Need $\frac{c}{2a} > 108$ MHz (No FM)

Need $\frac{c}{2a} < 162.4$ MHz (ALL NOAA)

→ $a_{min} = \frac{c}{2 \times 162 \text{MHz}} = \frac{3\times10^8 \text{ m/s}}{2\times162\times10^6} = \frac{300 \text{ m}}{324} = 0.926$ m

c) (8 pts) What is the maximum possible value of dimension a of the equivalent waveguide?

→ $a_{max} = \frac{c}{2\times108 \text{ MHz}} = \frac{3\times10^8 \text{ m/s}}{2\times108\times10^6} = \frac{300}{216} = 1.389$ m

d) (4 pts) If, instead of going straight down, the cave meanders a bit off-center on a scale which is long compared to the cross-sectional scale a of the cave, would you expect the reception quality in the NOAA band to get substantially worse (assuming a remains constant) as the spelunker descends into the cave? Discuss.

NOAA band is in a propagating state.
So no worsening is expected (unless the cave gets even narrower)