$\mu = 61.9/75$, $\sigma = 8.6$, med $= 64$

ECE 350 | Fields and Waves II | Spring 2019

University of Illinois

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Exam 2

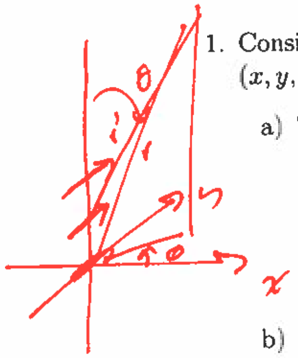
Fri, March 15, 2019 — Noon-12:55 PM

Name:	Solution
Section:	12 Noon

Please clearly PRINT your name in CAPITAL LETTERS and circle your section in the above boxes.

This is a closed book exam. You are allowed to bring two sheet of notes — both sides of the sheet may be used — and a calculator. Please show all your work and make sure to include your reasoning for each answer. All answers should include units wherever appropriate and simplified as much as possible.

Problem 1 (25 points)	
Problem 2 (25 points)	
Problem 3 (25 points)	
TOTAL (75 points)	



1. Consider a 1D N-element "phased antenna array" of $\hat{y}$-polarized short dipoles with element locations $(x, y, z) = (0, 0, nd)$ and input currents $I_n = 1\angle n\alpha$ A, with n in the interval $0, \cdots, N-1$.

a) The A.F. for far-field calculations can be expressed as $1 + w + w^2 + \cdots + w^{N-1}$. What is w in terms of d, α, k, and θ?

$$A.F. = 1 + \underbrace{e^{j(kd\cos\theta + \alpha)}}_{w} + w^2 + \ldots + w^{N-1} \qquad W = e^{j(kd\cos\theta+\alpha)}$$

b) What is the maximum possible value of the magnitude of the A.F. for this array of $N = 16$?

$$|A.F.|_{max} = 16$$

c) This A.F. will have a peak magnitude for directions θ for which $w = 1$ — for an arbitrary d for which α does the A.F. peak at $\theta = 90°$?

$\alpha = 0$ if peak is for $\theta = 90° \Rightarrow \cos\theta = 0$.

d) If $d = \lambda/4$, determine α with the smallest magnitude such that the A.F. peaks at a value of 16 in the direction $\theta = 0°$ for an "endfire" performance.

Need $W = \frac{2\pi}{\lambda}\frac{\lambda}{4}\underbrace{\cos 0°}_{1} + \alpha = 0 \Rightarrow \alpha = -\frac{\pi}{2}$ rad

e) For the endfire array in (d), for which $\angle w$ does A.F. exhibit its "first null", explain carefully? Hint: this depends on N !!!

forbidden

correct choice

$$W = e^{j\frac{\pi}{2}(\cos\theta - 1)} \Rightarrow \text{Need } \angle W = \frac{\pi}{2}(\cos\theta - 1) = \pm\frac{2\pi}{N}$$

$$\Downarrow \quad \underbrace{\cos\theta - 1}_{-\frac{\theta^2}{2}} = \pm\frac{4}{N} \Rightarrow \theta^2 = \frac{8}{N}$$

f) If the length of the endfire array in (d) were quadrupled, by what factor would the HPBW of the array change? — explain!

$N \to 4N$

$\theta \to \frac{\theta}{2}$ HPBW would drop by a factor of 2.

2. This problem has two independent parts:

a) A high directivity 2D circularly shaped antenna array mounted on a geostationary satellite about 36,000 km above central China is illuminating a circular region with a radius of 2,000 km with its main beam. The antenna is designed such that the solid angle of its main beam matches the solid angle of the circular region centered about central China as seen from the geostationary satellite. Answer the following questions:

i. (4 pts) What is the solid angle Ω_o of the antenna array? Explain.

$$\Omega_o = \frac{\pi a^2}{r^2} = \pi\left(\frac{2000}{36000}\right)^2 = \frac{\pi}{18^2} \text{ ster}$$

ii. (4 pts) What is the directivity D of the antenna array? Explain.

$$D = \frac{4\pi}{\Omega_o} = \frac{4\pi}{\pi/18^2} = 4\times 18^2 = 36^2 = 1296$$

iii. (4 pts) What would be the physical area A_{phys} of the antenna array for an operation wavelength $\lambda = \frac{1}{6}$ m? Here assume that directivity D and A_{phys} array are related same as in a filled square shaped 2D array.

$$D = \frac{4\pi}{\lambda^2} A_{phy} \Rightarrow A_{phy} = \frac{D\lambda^2}{4\pi} = \frac{(36/6)^2}{4\pi} = \frac{36}{4\pi} = \frac{9}{\pi} \text{ m}^2$$

~ 2.86

b) Consider a non-uniform plane wave with phasor $e^{-j\mathbf{k}\cdot\mathbf{r}}$ observed in free space having a complex valued wave vector $\mathbf{k} = 2\pi\hat{x} + j\pi\hat{y}$ 1/m. Answer the following questions:

i. (4 pts) In which direction does the non-uniform plane wave with phasor $e^{-j\mathbf{k}\cdot\mathbf{r}}$ decays exponentially in magnitude, $\pm x$ or $\pm y$? Explain.

$$e^{-j(2\pi x + j\pi y)} = \underbrace{e^{\pi y}}\, e^{-j2\pi x}$$

decays exponentially in $-\hat{y}$ direction.

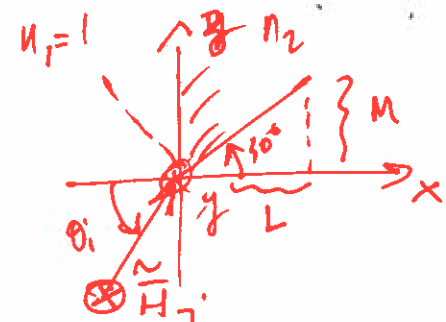
ii. (4 pts) What is the wavelength of the non-uniform plane wave $e^{-j\mathbf{k}\cdot\mathbf{r}}$?

$$k_x = \frac{2\pi}{\lambda_x} \Rightarrow \lambda_x = \lambda = 1 \text{ m}$$

iii. (5 pts) What is the frequency ω of the wave with phasor $e^{-j\mathbf{k}\cdot\mathbf{r}}$? Hint: ω and $\mathbf{k} = 2\pi\hat{x} + j\pi\hat{y}$ obey the TEM wave dispersion relation $\mathbf{k}\cdot\mathbf{k} = \frac{\omega^2}{c^2}$ for free space.

$$\bar{k}\cdot\bar{k} = (2\pi)^2 + j^2\pi^2 = 3\pi^2 = \frac{\omega^2}{c^2} \Rightarrow \omega = \sqrt{3}\pi c$$

$$= 3\sqrt{3}\pi\times 10^8 \ \frac{\text{rad}}{\text{sec}}$$



3. Consider a TM polarized TEM wave with magnetic field intensity vector

$$\tilde{\mathbf{H}}_i = \hat{y}\frac{1}{120\pi}e^{-j\mathbf{k}_i\cdot\mathbf{r}}\ \frac{\text{A}}{\text{m}}$$

propagating in air incident from the left on $x = 0$ plane beyond which there is a perfect dielectric with some refractive index n_2. The angle of incidence, $\theta_i = 60°$, of the TM polarized wave happens to be the Brewster's angle. Also $|\mathbf{k}_i| = 2\pi$ rad/m. Answer the following questions:

a) (4 pts) Given that $\cos 60° = \frac{1}{2}$, what is the vector $\mathbf{k}_i$ explicitly.

$$\bar{k}_i = 2\pi\left(\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z}\right) = 2\pi\left(\hat{x}\tfrac{1}{2} + \hat{z}\tfrac{\sqrt{3}}{2}\right) = \pi\left(\hat{x} + \hat{z}\sqrt{3}\right)\ \frac{\text{rad}}{\text{m}}$$

b) (4 pts) What is the angle of transmission θ_t?

$\theta_t = 30°$ so that $\theta_i + \theta_t = 90°$

$\frac{1}{120\pi}$ $\pi\sqrt{3}$

c) (4 pts) If we express the transmitted magnetic field intensity phasor as $\tilde{H}_t = \hat{y}Ke^{-j(Lx+Mz)}$ what are the numerical values of K and M?

$K = \frac{1}{120\pi}$, $M = \sqrt{3}\pi$ $\left(\frac{M}{L} = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \Rightarrow L = \sqrt{3}M = \sqrt{3}\sqrt{3}\pi = 3\pi\right)$

d) (4 pts) Using Snell's Law $n_1 \sin\theta_i = n_2 \sin\theta_t$ determine n_2.

$$\underbrace{n_1}_{1}\underbrace{\sin\theta_i}_{\frac{\sqrt{3}}{2}} = n_2\underbrace{\sin\theta_t}_{\frac{1}{2}} \Rightarrow n_2 = \sqrt{3}$$

e) (4 pts) Given n_2 from part (d), what are $|\mathbf{k}_t|$ and constant L defined in part (c)?

$|k_t| = |k_i|\,n_2 = 2\pi\sqrt{3}$ rad/m

$k_t^2 = L^2 + M^2 = (2\pi)^2 3 = L^2 + 3\pi^2$
$\Leftrightarrow L^2 = 9\pi^2 \Rightarrow L = 3\pi$

f) (2 pts) Given n_2 from part (d), what is the intrinsic impedance η_2 in the second medium?

$$\eta_2 = \frac{\eta_0}{n_2} = \frac{120\pi}{\sqrt{3}}\ \Omega$$

g) (3 pts) What is the magnitude $|\tilde{E}_t|$ of the transmitted electric field vector?

$$|H_t| = \frac{1}{120\pi}\ \text{A/m} \Rightarrow |E_t| = |H_t|\,\eta_2 = \frac{1}{\cancel{120\pi}}\cdot\frac{\cancel{120\pi}}{\sqrt{3}} = \frac{1}{\sqrt{3}}\ \text{V/m}$$