$\mu = 60.35$, $\sigma = 11.52$, med = 64.

ECE 350 | Fields and Waves II | Fall 2019

University of Illinois | Kudeki

Exam 2

Fri, Oct 25, 2019 — Noon-12:55 PM

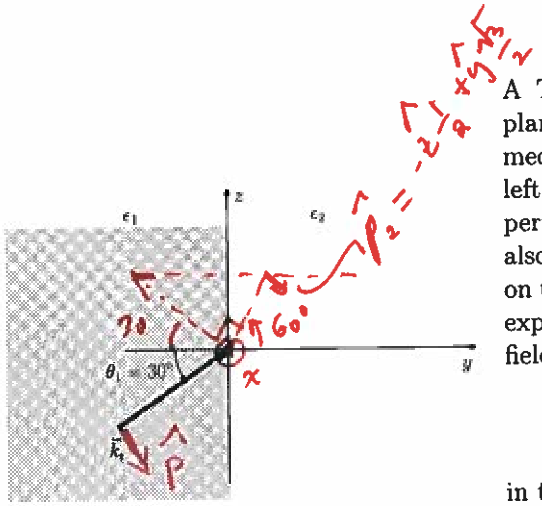
Name:	Solution.
Section:	12 Noon

Please clearly PRINT your name in CAPITAL LETTERS and circle your section in the above boxes.

This is a closed book exam, calculator allowed. You are allowed to bring two sheet of notes — both sides of the sheet may be used. Please show all your work and make sure to include your reasoning for each answer. All answers should include units wherever appropriate.

Problem 1 (25 points)	
Problem 2 (25 points)	
Problem 3 (25 points)	
TOTAL (75 points)	

1.



A TM polarized TEM wave is incident on $y = 0$ plane separating medium 1 with permittivity ϵ_1 from medium 2 with permittivity $\epsilon_2 = \epsilon_o$ as shown on the left. The permeability of both sides is the free space permeability μ_o and the *angle of incidence* $\theta_1 = 30°$ is also the **Brewster angle** in medium 1. The diagram on the left is also depicting the incident wavevector $\mathbf{k}_i$ explicitly. Also it is known that the incident electric field phasor is

$$\tilde{\mathbf{E}}_i = \hat{p}e^{-j\pi(\frac{\sqrt{3}}{2}y+\frac{1}{2}z)}\ \frac{\text{V}}{\text{m}}$$

in terms of some unit vector $\hat{p}$.

$\hat{k} = \hat{y}\frac{\sqrt{3}}{2} + \frac{1}{2}\hat{z}$

a) (2 pts) Given that $E_{zi} < 0$ at $(x, y, z) = (0, 0, 0)$, determine $\hat{p}$.

$\hat{p} = -\hat{z}\frac{\sqrt{3}}{2} + \hat{y}\frac{1}{2}$ // by inspection such that $\hat{p}\cdot\hat{k} = 0$

b) (2 pts) What is the *angle of transmission* θ_2?

$\theta_2 = 60°$ so that $\theta_1 + \theta_2 = 90°$

c) (2 pts) Draw the transmitted wave vector $\mathbf{k}_t$ in the above diagram in correct proportions and orientation with respect to $\mathbf{k}_i$.

d) (4 pts) Determine ϵ_1.

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ ($\sin\theta_1 = \frac{1}{2}$, $n_2 = 1$, $\sin\theta_2 = \frac{\sqrt{3}}{2}$) $\Rightarrow n_1 = \sqrt{3} \Rightarrow \varepsilon_1 = 3\varepsilon_0$ //

e) (3 pts) What is the frequency of the incident field in MHz units?

$k = \pi = \frac{\omega}{c/n} \Rightarrow \omega = 2\pi f = \pi\frac{c}{n} \Rightarrow f = \frac{c}{2n} = \frac{300\times10^6}{2\sqrt{3}} = \frac{150}{\sqrt{3}}$ MHz

f) (3 pts) What is the *incident magnetic field intensity phasor* $\tilde{\mathbf{H}}_i$?

$\bar{H}_i = -\hat{x}\frac{\sqrt{3}e^{-j\pi(\frac{\sqrt{3}}{2}y+\frac{1}{2}z)}}{120\pi}$ A/m

$\eta_1 = \frac{\eta_0}{n} = \frac{120\pi}{\sqrt{3}}$

g) (4 pts) What is the *transmitted magnetic field intensity phasor* $\tilde{\mathbf{H}}_t$?

$k_1 = \frac{\omega}{c}n_1 = \pi$

$\bar{H}_t = -\hat{x}\frac{\sqrt{3}e^{-j\frac{\pi}{\sqrt{3}}(\frac{1}{2}y+\frac{\sqrt{3}}{2}z)}}{120\pi}$ A/m

$k_2 = \frac{\omega}{c} = \frac{\pi}{n_1} = \frac{\pi}{\sqrt{3}}$

h) (5 pts) What is the *transmitted electric field phasor* $\tilde{\mathbf{E}}_t$?

$\bar{E}_t = \hat{p}_2\sqrt{3}\,e^{-j\frac{\pi}{\sqrt{3}}(\frac{1}{2}y+\frac{\sqrt{3}}{2}z)}$ V/m //

$\hat{p}_2 = -\hat{z}\frac{1}{2} + \hat{y}\frac{\sqrt{3}}{2}$

2. Consider a **TEM wave** propagating in a **good conductor** described by a phasor electric field

$$\tilde{\mathbf{E}} = \hat{y}e^{-j\mathbf{k}\cdot\mathbf{r}} \frac{\text{V}}{\text{m}}$$

such that $\mathbf{k}\cdot\mathbf{k} = k^2 = \omega^2\mu_o(\epsilon + \sigma/j\omega) \approx -j\omega\mu_o\sigma$ with $\omega\mu_o\sigma = 10^6\ \text{m}^{-2}$.

a) (4 pts) What is the value of k_y.

$k_y = 0$ because $\bar{E}\cdot\bar{k} = 0$.

b) (6 pts) Given the result of (a), and a constraint $k_x = 0$, determine the numerical value of k_z.

If $k_x = 0$ & $k_y = 0$, then

$\bar{k}\cdot\bar{k} = k_z^2 \cong -j\omega\mu_o\sigma$ ($\omega\mu_o\sigma = 10^6$) $\Rightarrow k_z = \frac{(1-j)}{\sqrt{2}}10^3$

$k_{zr} = k_{zi} = \frac{10^3}{\sqrt{2}}\ \frac{\text{rad}}{\text{m}}$

c) (3 pts) Given the results of (a) and (b) what is the penetration depth of the field in mm units?

$\delta = \frac{1}{k_{zr}} = \sqrt{2}\times 10^{-3}\ \text{m} = \sqrt{2}\ \text{mm}.$

d) (3 pts) Given the results of (a) and (b) what is the wavelength of the field in mm units?

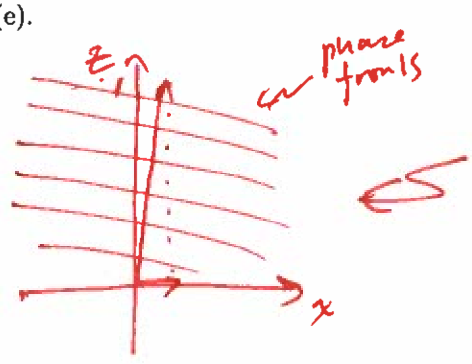
$\lambda = \frac{2\pi}{k_{zr}} = 2\pi\delta = 2\pi\sqrt{2}\ \text{mm}$

e) (6 pts) Suppose some boundary phase matching condition places a constraint $k_x = 1$ rad/m for the region $z > 0$. What would be the simplified phasor expression for $\tilde{\mathbf{E}}$ in $z > 0$ region in that case?

$k_x^2 + k_z^2 \cong -j\,10^6$ ($k_x = 1$) $\Rightarrow k_z \cong \frac{(1-j)}{\sqrt{2}}10^3\ \frac{\text{rad}}{\text{m}}$

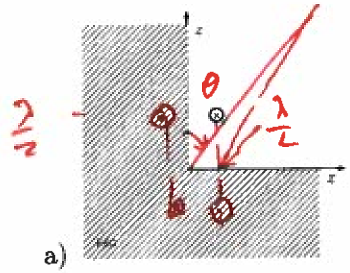
$\tilde{E} = \hat{y}\, e^{-jx}\, e^{-j\frac{(1-j)}{\sqrt{2}}10^3 z}$

f) (3 pts) Describe in words the structure of the field variation implied by the phasor found in part (e).



what we have is a damped plane wave with very shallow tilted phase fronts seen on the left

3. Two parts of this question are independent:

a) 

Consider a $\hat{y}$ polarized short dipole antenna located at $(x, y, z) = (\lambda/4, 0, \lambda/8)$ next to a PEC corner reflector as shown in the diagram depicted on the left. The dipole is indicated by the "tail view" of an arrow pointing in $+y$ direction.

i. (5 pts) Indicate the locations and directions of image currents (with tail and head views of arrows pointing in $\pm y$ directions) that can be used to model the radiation fields of the dipole into the $0 \leq \theta \leq 90°$ sector. Explain why!

In order to maintain $E_{tan} = 0$ on both $z=0$ and $x=0$ surfaces.

ii. (10 pts) Determine the array factor A.F. of the system of currents in (i) for the sector $0 \leq \theta \leq 90°$ for an observer on xz-plane. Simplify your expression as much as you can. **Hint:** this is a 1D array (x-directed) elements of which are also 1D arrays (z-directed) ... use "pattern multiplication" of the two 1D A.F.'s

$$A.F. = E_{el}\left(e^{jk\frac{\lambda}{4}\sin\theta} - c.c.\right)\left(e^{jk\frac{\lambda}{2}\cos\theta} - c.c\right) = (2j)^2 \sin(\pi\cos\theta)\sin\left(\frac{\pi}{2}\sin\theta\right)$$

iii. (5 **bonus** pts) For which value of θ is |A.F.| maximized in the sector $0 \leq \theta \leq 90°$?

Using calculator $\theta_{peak} \cong 1.067$ rad $\cong 61.13°$

b) Two space ships are blasting away from Earth in the same direction at high but unequal speeds. The trailing space ship of Major Tom moving away from Earth with a speed of $v = 0.6c$ sends pulses with 3 GHz which are echoed back to Major Tom from Captain Tina's ship racing ahead at a Doppler shifted frequency of 2.99 GHz.

i. (5 pts) What is the Doppler shifted frequency f_E of Major Tom's 3 GHz emissions detected on Earth? 3 GHz (M.T.)

E   Tom   TINA

$\vec{v} = 0.6c$

$$f_E = f_{Tom}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} = 3\text{ GHz}\sqrt{\frac{1-0.6}{1+0.6}} = 1.5\text{ GHz}$$

$\frac{0.4}{1.6} = \frac{1}{4}$

ii. (5 pts) What is the speed of Captain Tina in the reference frame of Major Tom in km/s units?

$$f''_{Tom} = f_{Tom}\frac{1-\frac{v}{c}}{1+\frac{v}{c}} \rightarrow \frac{2.99}{3} = \frac{1-\frac{v}{c}}{1+\frac{v}{c}} \Rightarrow 2.99 + 2.99\frac{v}{c} = 3 - 3\frac{v}{c}$$

$5.99\frac{v}{c} = 0.01$

$\frac{v}{c} = \frac{1}{599}$

$v \cong \frac{300\times10^6}{600} = 0.5\times10^6 \frac{m}{s} = 500 \frac{km}{s}$