$\mu = 63.6/75$, $\sigma = 7.43$, $\mu_{med} = 64.50$

ECE 350 **Fields and Waves II** **Spring 2019**

University of Illinois **Kudeki**

Exam 1

Monday, Feb 18, 2019 — Noon-12:55 PM

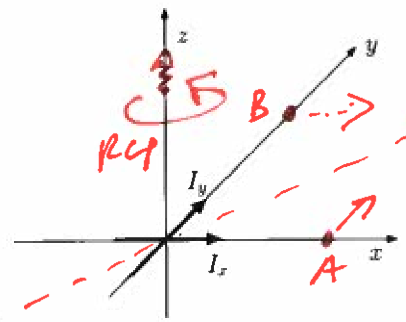
Name:	Solution
Section:	12 Noon

Please clearly PRINT your name in CAPITAL LETTERS.

This is a closed book exam. You are allowed to bring one sheet of notes — both sides of the sheet may be used. Please show all your work and make sure to include your reasoning for each answer. All answers should include units wherever appropriate.

Problem 1 (25 points)	
Problem 2 (25 points)	
Problem 3 (25 points)	
TOTAL (75 points)	

1.



A radiating system consists of a pair of short dipoles of identical lengths $L = \lambda/100$ which are configured along the x- and y-axes, respectively, as shown on the left. Their input current phasors are labeled as I_x and I_y, respectively and as shown in the figure. The ~~radiation~~ electric field phasor at $(x, y, z) = (100\lambda, 0, 0)$ is given as $\hat{y}2e^{-j200\pi}$ V/m and the time averaged Poynting vector magnitudes (Poynting flux) at $(x, y, z) = (100\lambda, 0, 0)$ and $(x, y, z) = (0, 100\lambda, 0)$ are identical. Answer the following questions:

a) (5 pts) What is the value of $|I_x/I_y|$? — explain your reasoning.

Same flux at A & B $\Rightarrow$ $|I_x/I_y| = 1$

b) (5 pts) If the phasor I_y is expressed as $I_o\angle\gamma$, I_o a positive real number, what would be the angle γ of I_y — explain your reasoning.

$$\bar{E}_A = \hat{y}\, 2 e^{-j200\pi} = j\, \cancel{I_y}\, k\left(\tfrac{L}{2}\right) \frac{e^{-jkr}}{4\pi r}(-\hat{y})$$

$I_o e^{j\gamma}$

If $\gamma = \frac{\pi}{2}$ rad Works out !!

c) (5 pts) If $I_x = I_y$, what is $\tilde{\mathbf{E}}(0, 200\lambda, 0)$? — explain your reasoning.

$\tilde{E}(0, 200\lambda, 0) = \hat{x}\, e^{-j400\pi}$ V/m because 200λ is twice the distance to 100λ

d) (5 pts) If $I_x = I_y$, what is $\tilde{\mathbf{E}}(100\lambda, 100\lambda, 0)$? — explain your reasoning.

$\hookrightarrow$ 0 because $I_{eff} = \sqrt{2}\, I_x$ along the dashed line

e) (5 pts) If the radiating system is producing a right circular polarized (RCP) radiation wave field at $(x, y, z) = (0, 0, 100\lambda)$ what is I_y in terms of I_x?

I_x should lead I_y by $90°$

$\hookrightarrow$ $\therefore I_y = -j I_x$

2. **Gain and directivity:**

a) (10 pts) An antenna has a gain function $G(\theta, \phi) = D\sin^2\theta \text{rect}(\frac{\phi}{W})$ over all $0 < \theta < \pi$ and $-\pi < \phi < \pi$. If directivity $D = 6$ what are W and the beam solid angle Ω_o. **Hint:** $\text{rect}(\frac{x}{W})$ equals 1 for $-W/2 < x < W/2$ and 0 otherwise.

Short dipole: $D = 1.5$, $W = 2\pi$

This antenna $D = 6.0$, $W = \frac{2\pi}{4} = \frac{\pi}{2}$

Also, $D\Omega_o = 4\pi \Rightarrow \Omega_o = \frac{4\pi}{6} = \frac{2}{3}\pi$ ster.

b) (15 pts) An antenna with a radiation resistance of $R_{rad} = 2$ ohms, directivity $D = 3/2$, and input current phasor of $\tilde{I}_o = \frac{1}{\sqrt{90}}$ A is being operated in vacuum.

i. What is the average radiated power P_{rad} of the antenna?

$$P_{rad} = \frac{1}{2} R_{rad} |\tilde{I}_o|^2 = \frac{1}{2} \cdot 2 \cdot \left(\frac{1}{\sqrt{90}}\right)^2 = \frac{1}{90} \text{ W}$$

ii. Calculate the maximum Poynting flux of the antenna radiation field at a distance of 100 m and give a reasonable approximate value for it in μW/m^2 units (without using a calculator)?

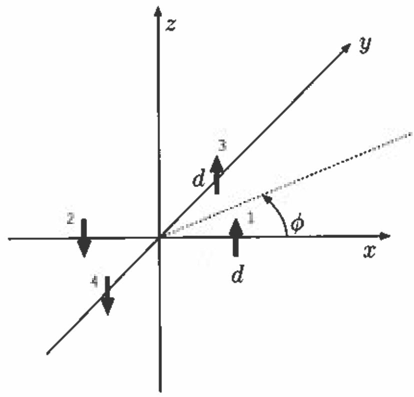
$$\frac{|E|^2}{2\eta_o} = \frac{P_{rad}}{4\pi r^2} G = \frac{\frac{1}{90}}{4\pi (100)^2} \frac{3}{2} = \frac{10^{-4}}{240\pi} = \frac{100}{240\pi} 10^{-6}$$

$$= \frac{10}{24\pi} \frac{\mu W}{m^2} = \frac{1}{2.4\pi} \cong \frac{1}{8} \frac{\mu W}{m^2}$$

iii. What is the corresponding electric field amplitude at the same location in mV/m units (make this an exact calculation)?

$$\frac{|E|^2}{240\pi} = \frac{10^{-4}}{240\pi} \Rightarrow |E| = 10^{-2} \frac{V}{m} = 10 \frac{mV}{m}$$

3.



Consider the four-element array of identical $\hat{z}$-polarized Hertzian dipoles placed at a distance d away from the origin on $\pm x$ and $\pm y$ axes as depicted on the left. All four dipoles are driven by identical input currents in the reference directions indicated by dipole arrows. The medium is free space.

a) (5 pts) Retarded potential phasor $\tilde{\mathbf{A}} = 0$ at $(x, y, z) = (0, 0, 100\lambda)$ — TRUE or FALSE ? Explain your choice.

1&2 cancel at the origin
3&4 also

b) (5 pts) Radiation electric field phasor $\tilde{\mathbf{E}}_r = 0$ at $(x, y, z) = (0, 0, 0)$ — TRUE or FALSE ? Explain your choice.

Same reason as (a)

c) (5 pts) Radiation magnetic field phasor $\tilde{\mathbf{H}}_r = 0$ at $(x, y, z) = (0, 0, 0)$ — TRUE or FALSE ? Explain your choice.

1 & 2 generate $\tilde{H} \propto -\hat{y}$
3 & 4 generate $\tilde{H} \propto \hat{x}$
)) Non-cancelling.

d) (5 pts) For this four-element antenna array the total retarded potential phasor $\tilde{\mathbf{A}} = 0$ for an observer on xy-plane at a radial distance $r \gg d$ and with an azimuth angle of either $\phi = 45°$ or $\phi = -45°$. Which azimuth, $\phi = 45°$ or $\phi = -45°$, could have a non-zero $\tilde{\mathbf{A}}$? Explain your answer carefully giving a reason why $\tilde{\mathbf{A}}$ could be non-zero.

$\tilde{A} \neq 0$ on $\phi = 45°$ is possible depending on d.

For $\phi = -45°$, $\tilde{A} = 0$ for all d: 1&4 cancel, so do 3 & 2.

e) (5 pts) If dipole 1 situated at $(x, y, z) = (d, 0, 0)$ is causing by itself a field $\tilde{\mathbf{E}}_r = \hat{z}C$ at a location $(x, y, z) = (r, 0, 0)$, such that $r \gg d$ and C some complex number, what would be the total radiation field $\tilde{\mathbf{E}}_{tot}$ of the four-element array at another location at $(x, y, z) = (2r, 0, 0)$, assuming that $d = \lambda/4$? Explain your answer carefully by describing how, under paraxial approximation, each of the dipoles (1, 2, 3, and 4) would be contributing to $\tilde{\mathbf{E}}_{tot}$.

for $C = 2\angle 0° = 2 + j0 = 2$:

$E_{tot} = \underbrace{\frac{2\hat{z}}{2}}_{\text{from 1}} + \underbrace{\frac{2\hat{z}}{2}}_{\text{from 2}} = 2\hat{z}\ \frac{V}{m}$ as 3&4 contributions cancel.

— because $2d = \lambda$