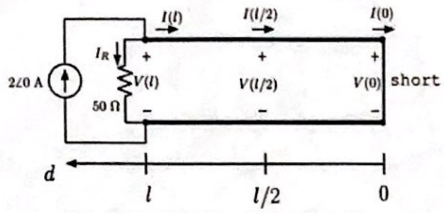
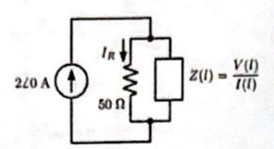
ECE 350 - Fa20, HE1 Name: NetID:

1. Consider a transmission line segment of propagation velocity $v = c = 3 \times 10^8$ m/s, characteristic impedance $Z_o = 50\,\Omega$, length l, and a *short* termination. As shown in figure (a) below, this "shorted stub" is connected *in parallel* with a 50 Ω **resistor** and an ideal **current source** $i(t) = \mathrm{Re}\{Ie^{j\omega t}\}$, where, $I = 2\angle 0$ A is the source current phasor and $\omega = 2\pi \times 10^8$ rad/s; figure (b) depicts an *equivalent circuit* in terms of input impedance of the transmission line at $d = l$, namely $Z(l) \equiv \frac{V(l)}{I(l)}$.



(a) Transmission line circuit



(b) Equivalent circuit

In answering the following questions assume that the circuit above is in sinusoidal steady-state:

a) (2 pts) What is the signal wavelength λ (in m) on the transmission line?

b) (2 pts) Since a "short termination" is located on the line at $d = 0$, what is the pertinent "boundary condition" involving the phasor $V(0)$ for all possible stub lengths l?

c) (2 pts) Does the transmission line support a *standing wave* in the above circuit for all values of non-zero l? Justify your answer.

d) (2 pts) What is the smallest non-zero value of l (in m) if phasor $I_R = 0$ A? Explain.

e) (2 pts) For l determined in part (d), what is phasor $I(l/2)$? Explain.

f) (2 pts) For l determined in part (d), is $V(l/2)$? **Hint:** first figure out $I(0)$ and then use the "current forcing formula" to get $V(l/2)$!

g) (2 pts) What is the smallest value of l if $I_R = 2\angle 0$ A?

h) (2 pts) Given that $V(l/2) = 0$, what is the smallest possible value of l? Explain

i) (2 pts) For l determined in part (h), what is $I(l)$? Explain.

See Smith Chart next

j) (2 pts) On a Smith Chart mark and label the normalized impedance $z(d) \equiv \frac{Z(d)}{Z_o}$ for $d = 0$ and $d = l$ if $l = 0.3\lambda$.

k) (2 pts) Using the Smith Chart show that the value of $y(0.3\lambda) = \frac{1}{z(0.3\lambda)}$ matches $z(0.55\lambda)$. Explain your reasoning.

l) (3 pts) Using the same Smith Chart determine $Z(0.3\lambda)$ on a TL if $z_L = 1 - j1$ and $Z_o = 50\,\Omega$. Show your work on the Smith Chart.

a) $\omega = 2\pi \times 10^8 = 2\pi f \Rightarrow f = 10^8$ Hz $\Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8 \text{ m/s}}{10^8 \text{ 1/s}} = 3$ m ///.

b) $V(0) = 0$!

c) Yes because $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$ //

d) $I_R = 0 \Rightarrow V(l) = 0 \Leftrightarrow$ short $\Rightarrow l = \frac{\lambda}{2} = 1.5$ m //

e) $I(l/2) = 0$

f) $I(0) = -j\frac{V(l/2)}{Z_0} \Rightarrow V(l/2) = j\underset{-2A}{I(0)}\underset{50\Omega}{Z_0} = -j100$ V //

$I(0) = -I(l) = -2A$

g) $I_R = 2A \Rightarrow I(l) = 0 \Leftrightarrow$ open $\Rightarrow l = \lambda/4 = 0.75$ m //

h) $V(l/2) = V(0) = 0 \Leftrightarrow l/2 = \lambda/2 \Rightarrow l = \lambda = 3$ m //

i) $V(l) = 0 \Rightarrow I_R = 0$ & $I(l) = 2A$ ///.

$z(d)\big|_{d=0} = 0 + j0$

$z(0.55\lambda) = 0 + j0.32 \cong \dfrac{1}{-j3.1}$

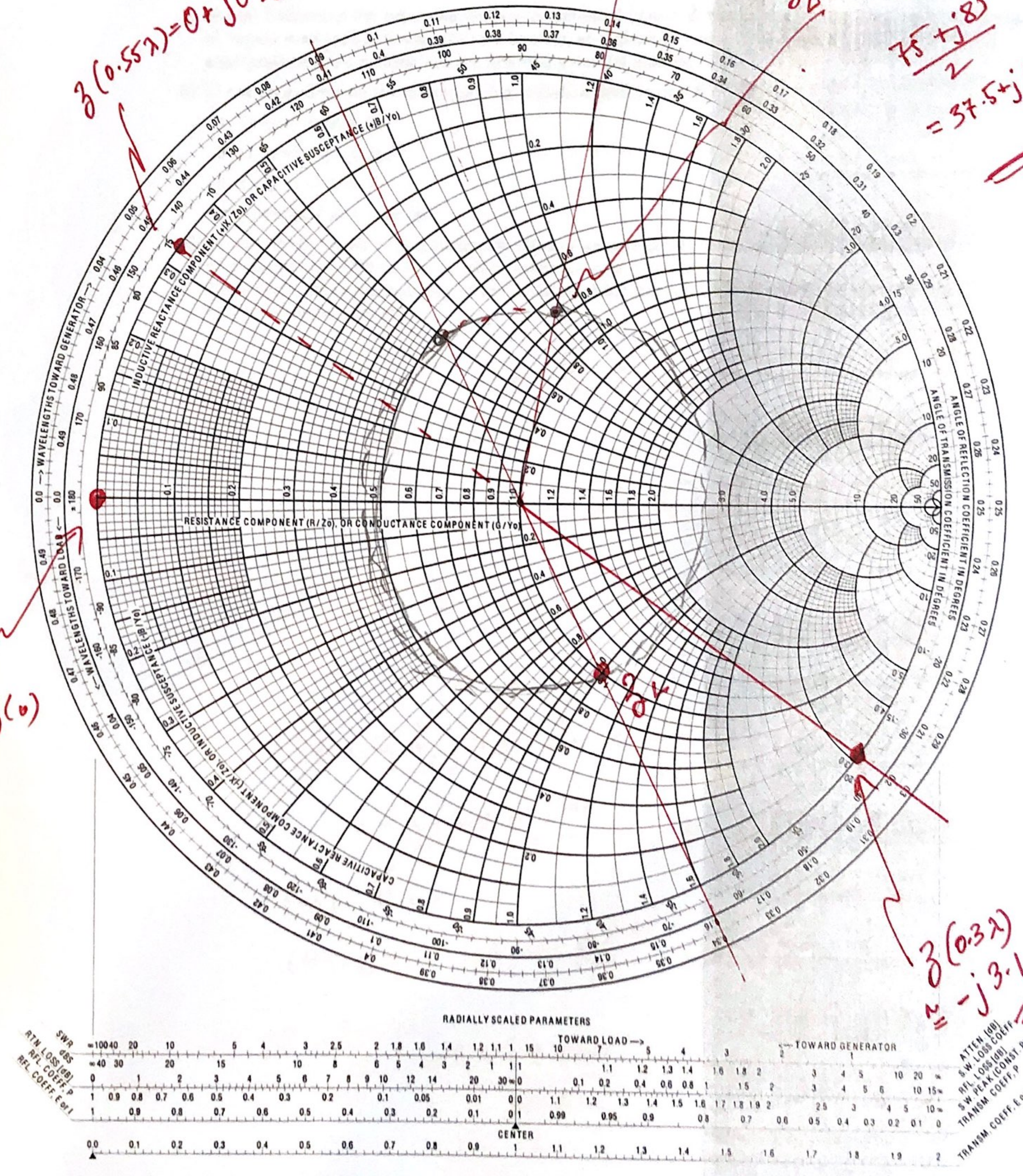
Smith Chart

$z_L \rightarrow 0.75 + j0.85$

$\downarrow \times 50\,\Omega$

$\dfrac{75 + j85}{2}$

$= 37.5 + j42.5\ \Omega$



$z(0)$

$z(0.3\lambda) \cong -j3.1$

2. Two identical Hertzian dipoles $I\Delta\ell$ are co-located at the origin aligned with the positive x- and y-axis, respectively. The strength of the dipoles is given as $I\Delta\ell = 1$ A.m, $k = 2\pi$ rad/m, and the dipoles are embedded in free space with $\eta_o = 120\pi\,\Omega$, $\mu_o = 4\pi \times 10^{-7}$ H/m.

   a) (5 pts) Determine the total retarded vector potential phasor $\tilde{\mathbf{A}}$ expression of the dipoles acting in unison evaluated at some arbitrary location at a distance r away from the origin. **Hint:** use superposition after figuring out the retarded potential phasors of the x- and y-polarized dipoles.

   b) (5 pts each) Express the corresponding *radiation field* phasor $\tilde{\mathbf{E}}$ of the dipoles at

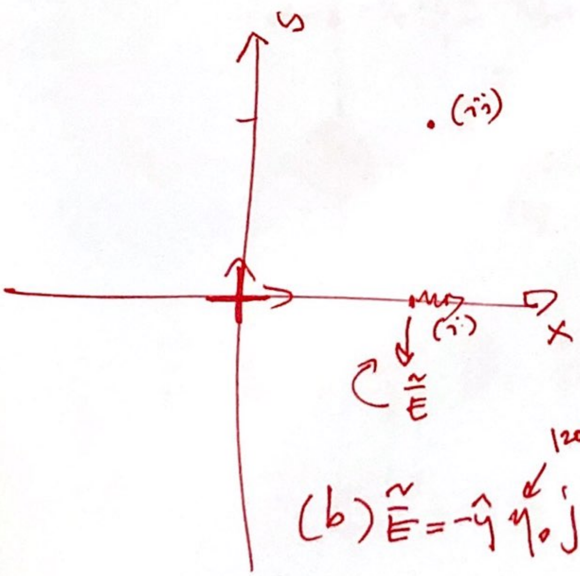
      i. $(x, y, z) = (d, 0, 0)$

      ii. $(x, y, z) = (d, d, 0)$

      iii. $(x, y, z) = (0, 0, d)$

      for distances $d \gg \lambda = 2\pi/k$.

   c) (5 pts) What would be the maximum value of average power density $|\langle \mathbf{E} \times \mathbf{H} \rangle|$ of the dipoles at a distance of 1000 m and at which locations (x, y, z) would these maxima would be found? Hint: think about the topology of the radiation fields leaving the antennas and take advantage of the results for part (b)



a) $\tilde{A} = (\hat{x}+\hat{y})\,\overset{4\pi\times10^{-7}}{\mu_o}\,\frac{\overset{1}{I\Delta\ell}}{4\pi r}\,e^{-jkr}$

$= (\hat{x}+\hat{y})\,\frac{10^{-7}}{r}\,e^{-j2\pi r}$ $\quad r \leftarrow 1000$

$= (\hat{x}+\hat{y})\,10^{-10}\ \frac{Wb}{m}$ //.

(b) $\tilde{E} = -\hat{y}\,\overset{120\pi}{\eta_o}\,j\,\overset{1}{I\Delta\ell}\ \text{"}\overset{1}{\sin\theta}\text{"}\ \overset{2\pi}{k}\,\frac{e^{-jkd}}{4\pi d}$

$= -\hat{y}\,j\,60\pi\,\frac{e^{-j2\pi d}}{d}$ V/m // (i)

(ii) $\tilde{E} = 0$

(iii) $\tilde{E} = (-\hat{x}-\hat{y})\,j\,60\pi\,\frac{e^{-j2\pi d}}{d}$ V/m //.

(c) $\frac{|E|^2}{2\eta_o} = \frac{\left(\sqrt{2}\times\frac{60\pi}{10^3}\right)^2}{240\pi} = \frac{2\times60\pi\times60\pi}{240\pi}\times10^{-6} = 30\pi\times10^{-6}\ \frac{W}{m^2}$ //.

3. A small loop antenna in free space and centered about the origin on the xy-plane is producing a (far-field) radiation electric field (in phasor notation)

$$\tilde{\mathbf{E}} = -\hat{x}\, 2\frac{e^{-jkr}}{r}\,\frac{\mathrm{V}}{\mathrm{m}}$$

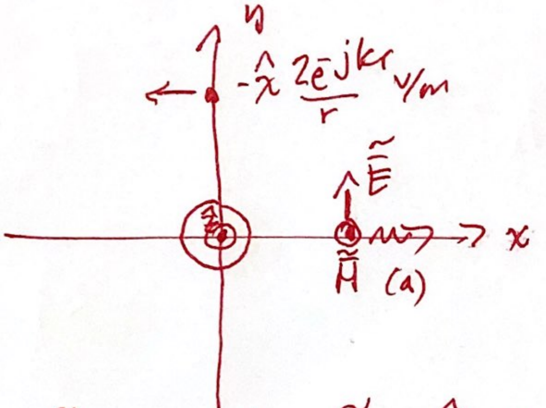
at an observer location $(x, y, z) = (0, r, 0)$ when it is driven with a loop current of $1\angle 0°$ A in counter-clockwise direction when viewed from above the xy-plane. Here r is some positive number indicating the distance of the observer from the origin.

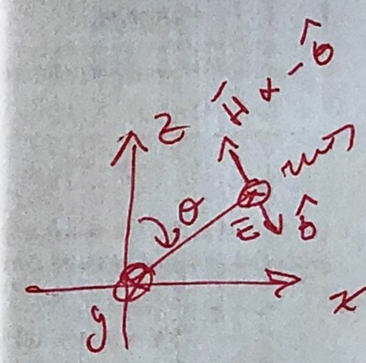
a) (5 pts) What are the phasors $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ at distance r but for $\theta = 90°$ and $\phi = 0°$?

b) (10 pts) What are the phasors $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ at distance r and $\phi = 0°$ for arbitrary θ?

c) (5 pts) What is the average power density $|\langle \mathbf{E} \times \mathbf{H} \rangle|$ at a radial distance r based on part (b)?

d) (5 pts) What is the total power radiated by this loop antenna. **Hint:** you need to do the $\int d\Omega r^2 |\langle \mathbf{E} \times \mathbf{H} \rangle|$ integral using the result of (c) and remembering that $\int_0^\pi d\theta \sin^3\theta = 4/3$ from class.





(a) $\tilde{E} = \hat{y}\,\frac{2e^{-jkr}}{r}\,\frac{V}{m}$, $\tilde{H} = \hat{z}\,\frac{1}{120\pi}\,\frac{2e^{-jkr}}{r}\,\frac{A}{m}$

(b) $\tilde{E} = \hat{y}\,\frac{2e^{-jkr}}{r}\sin\theta\,\frac{V}{m}$, $\bar{H} = -\hat{\theta}\,\frac{\sin\theta}{120\pi}\,\frac{2e^{-jkr}}{r}\,\frac{A}{m}$

(c) $|\langle \bar{E} \times \bar{H} \rangle| = \frac{|E|^2}{2\eta_0} = \frac{\frac{4\sin^2\theta}{r^2}}{240\pi} = \frac{\sin^2\theta}{60\pi r^2}\,\frac{W}{m^2}$

d) $P_r = \int d\Omega\, \cancel{r^2}\,\frac{\sin^2\theta}{60\pi \cancel{r^2}} = \frac{2\cancel{\pi}}{60\cancel{\pi}}\underbrace{\int_0^\pi d\theta \sin\theta \sin^2\theta}_{4/3} = \frac{4}{90} = \frac{2}{45}$ W

0.0444 W