$\mu = 66/75$, $\sigma = 9.68$, med $= 69$

ECE 350 Fields and Waves II Fall 2019

University of Illinois Kudeki

Exam 1

Monday, Feb 18, 2019 — Noon-12:55 PM

Name:	Solution
Section:	12 Noon

Please clearly PRINT your name in CAPITAL LETTERS.

This is a closed book exam. Calculators are allowed. You are allowed to bring one sheet of notes — both sides of the sheet may be used. Please show all your work and make sure to include your reasoning for each answer. All answers should include units wherever appropriate.

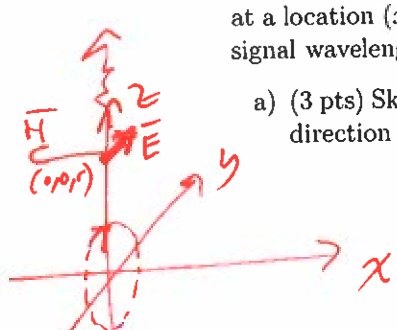
Problem 1 (25 points)	
Problem 2 (25 points)	
Problem 3 (25 points)	
TOTAL (75 points)	

1. **Radiation fields of a loop antenna:** A small loop antenna centered about the origin carrying a current with amplitude I_o has a radiation null (i.e., $\tilde{\mathbf{E}} = 0$) along the x-axis and is generating a radiation electric field with a phasor

$$\tilde{\mathbf{E}} = 2\times10^{-3}e^{-jkr}\hat{y}\,\frac{\mathrm{V}}{\mathrm{m}}$$

at a location $(x, y, z) = (0, 0, r)$ on the z-axis with $r = 10^3$ m, much larger than the loop radius a and signal wavelength $\lambda = 2\pi/k$.

a) (3 pts) Sketch the orientation of the loop antenna with respect to xyz-axes and mark the reference direction of I_o.



Loop on y-z plane

b) (3 pts) Determine the radiation magnetic field intensity phasor $\tilde{\mathbf{H}}$ at the same location $(x, y, z) = (0, 0, r)$.

$$\tilde{H} = -\hat{x}\frac{E_y}{\eta_o} = -\hat{x}\frac{2\times10^{-3}}{120\pi}e^{-jkr} = -\hat{x}\frac{10^{-3}}{60\pi}e^{-jkr}\,\frac{A}{m}$$

c) (3 pts) Determine the time averaged Poynting vector $\langle\mathbf{S}\rangle$ at the same location.

$$\langle \bar{E}\times\bar{H}\rangle = \frac{|\tilde{E}|^2}{2\eta_o}\hat{z} = \frac{4\times10^{-6}}{240\pi}\hat{z} = \frac{10^{-6}}{60\pi}\hat{z}\,\frac{W}{m^2}$$

$\approx 5.3\times10^{-9}$

d) (8 pts) Determine the average radiated power P_{rad} of the loop antenna (remembering that dipole and loop antennas have identical directivities of $D = 3/2$).

$$\frac{10^{-6}}{60\pi} \rightarrow \langle \bar{E}\times\bar{H}\rangle = \frac{P_{rad}}{4\pi\,10^6}\,\frac{3}{2} \Rightarrow P_{rad} = \frac{4\pi}{60\pi}\,\frac{2}{3} = \frac{2}{45}\ W$$

≈ 0.044 W

e) (3 pts) Determine the antenna radiation resistance R_{rad} if the loop current amplitude I_o is 1 A.

$$\frac{2}{45} \rightarrow P_{rad} = \frac{1}{2}R_{rad}|I|^2 \;(1A) \Rightarrow R_{rad} = \frac{4}{45}\,\Omega$$

$\approx 0.088\ \Omega$

f) (5 pts) Determine the radiation electric field phasor $\tilde{\mathbf{E}}$ at location $(x, y, z) = (0, r/2, 0)$.

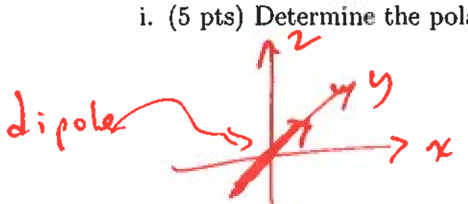
$$\tilde{E}(0,\tfrac{r}{2},0) = 2\times(2\times10^{-3}(-\hat{z}))\times10^{-jk\frac{r}{2}} = -\hat{z}\,4\times10^{-3}e^{-jk\frac{r}{2}}\ V/m$$

2. **Gain, directivity, solid angle:**

{ y-polarized short dipole

a) Consider a dipole antenna with a gain function $G(\theta, \phi) = K \sin^2 \theta_y = K(1 - \sin^2 \theta \sin^2 \phi)$ over all θ and ϕ, where K is some constant.

i. (5 pts) Determine the polarization axis of the dipole and show it with a simple sketch.



ii. (5 pts) What would be the numerical value of K — explain your reasoning briefly (no complicated math is needed!)

short dipoles have directivities of $D = \frac{3}{2}$

$\therefore K = \frac{3}{2}$ & $G(\theta,\phi) = \frac{3}{2}\sin^2\theta_y$ // for y-pol. dipole

b) An antenna with directivity $D = \pi$ is radiating an average power of 4 W.

i. (5 pts) If the antenna is operated in vacuum what would be the maximum Poynting flux of its radiation field at a distance of 1 km in μW/m^2 units?

$D = \pi$, $P_{rad} = 4W$, $r = 10^3$ m

$\therefore \frac{|\tilde{E}|^2}{2\eta_o} = \frac{P_{rad}}{4\pi r^2} D = \frac{4}{4\pi 10^6}\pi = 10^{-6} \frac{W}{m^2} = 1 \frac{\mu W}{m^2}$ //

ii. (5 pts) What would be the corresponding electric field amplitude at the same location in mV/m unit?

$\frac{|\tilde{E}|^2}{2\eta_o} = 10^{-6} \Rightarrow |\tilde{E}|^2 = 240\pi \times 10^{-6}$

$\Rightarrow |\tilde{E}| = \sqrt{240\pi} \times 10^{-3}$ V/m $= \sqrt{240\pi}$ $\frac{mV}{m}$ //

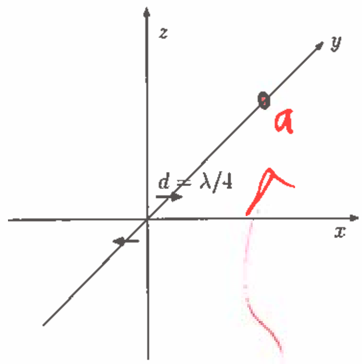
≈ 27.45

iii. (5 pts) Would the beam of the antenna have larger or smaller solid angle Ω_o than a short dipole beam - why? What are the relevant Ω_o values?

$D\Omega_o = 4\pi \Rightarrow$ Short dipole: $\Omega_o = \frac{4\pi}{3/2} = \frac{8\pi}{3} \approx 8$ ster.

This antenna $\Omega_o = \frac{4\pi}{\pi} = 4$ ster.

↑ smaller solid angle for this antenna compared to short dipole



Consider a system of two x-polarized short dipoles placed on the y-axis at $y = \pm d = \pm\lambda/4$ as shown on the left. Both dipole lengths are $L \ll \lambda$ and both dipoles carry the same non-zero input currents except for a 180 degree phase shift, which is why the current reference directions of the dipoles are indicated by opposing arrows in the figure.

3.

a) (5 pts) Vector potential $\tilde{\mathbf{A}} = 0$ at $(x, y, z) = (r, 0, 0)$ for all $r > d$. Would it also be true that $\tilde{\mathbf{A}} = 0$ at $(x, y, z) = (0, r, 0)$ for all l $r > d$? YES or NO? Circle one and explain briefly.

see here

at point (a) two $\tilde{A}$'s dont cancel out because they point in the same direction because of $\frac{\lambda}{2}$ extra travel distance.

b) (5 pts) $\tilde{\mathbf{H}} = 0$ at $(x, y, z) = (0, 0, 0)$ — TRUE or FALSE? Circle one and explain briefly.

By right hand rule $\bar{H}$ is in $-\hat{z}$ direction from both dipoles (at the origin)

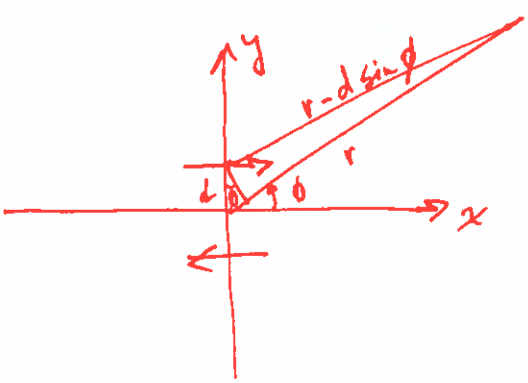
c) (5 pts) The time-average Poynting vector $\langle \mathbf{E} \times \mathbf{H} \rangle = 0$ at $(x, y, z) = (0, 0, 0)$ — TRUE or FALSE? Circle one and explain briefly.

$\bar{E}_{tot} = 0$ at the origin because of cancellation and that makes $\langle \bar{E}_{tot} \times \bar{H}_{tot} \rangle = 0$.

d) (5 pts) $\tilde{\mathbf{E}} = 0$ at $(x, y, z) = (0, r, 0)$ — TRUE or FALSE? Circle one and explain briefly.

same reason as in part (a).

e) (5 pts) What is the array factor (A.F.) on the xy-plane as a function of azimuth angle ϕ? Simplify your result as much as you can and find its magnitude $|\text{A.F.}|$ for $\phi = 90°$.



$\bar{E}_{tot} = E_0 \{ e^{+jkd\sin\phi} - e^{-jkd\sin\phi} \}$

because the dipoles are pointing in opposite direction

$\text{A.F.} = 2j \sin(kd\sin\phi) = 2j\sin\left(\frac{\pi}{2}\sin\phi\right)$ ($k = 2\pi/\lambda$, $d = \lambda/4$)

$|\text{A.F.}|_{\phi=90°} = \left|2j\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right)\right)\right| = |2j| = 2$