39 Lossy lines

- *Lossless* TL's we have been studying so far are idealizations of *real* TL's which are invariably **lossy**.

  – Here, we are making reference to ***Ohmic*** energy losses in the conducting wires of the TL, as well as to losses in the imperfect dielectric separating the two conductors.

- The effect of wire losses in TL's is modeled by adding a $\Delta z\mathcal{R}$ resistance in series with $\Delta z\mathcal{L}$ inductor in the equivalent circuit model of an infinitesimal ($\Delta z \ll \lambda$) TL section as shown in the margin.

- In addition, a shunt conductance $\Delta z\mathcal{G}$ in parallel with capacitance $\Delta z\mathcal{C}$ accounts in the lossy model for dielectric losses.

- While the phasor form of telegrapher's equations for a lossless TL is

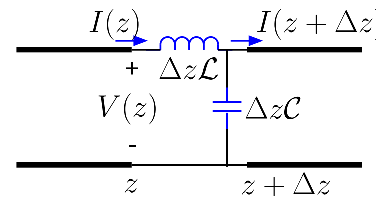
$$-\frac{\partial V}{\partial z} = j\omega\mathcal{L}I \quad \text{and} \quad -\frac{\partial I}{\partial z} = j\omega\mathcal{C}V,$$

  for lossy lines — where impedance per unit length $j\omega\mathcal{L}$ must be replaced by $j\omega\mathcal{L} + \mathcal{R}$ and conductance per unit length $j\omega\mathcal{C}$ by $j\omega\mathcal{C} + \mathcal{G}$ — the equations take the form

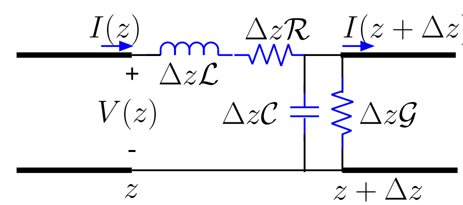
$$-\frac{\partial V}{\partial z} = (j\omega\mathcal{L} + \mathcal{R})I \quad \text{and} \quad -\frac{\partial I}{\partial z} = (j\omega\mathcal{C} + \mathcal{G})V.$$

- We will next show that

IDEAL LOSSLESS T.L.:



REALISTIC LOSSY T.L.:



Using perturbation theory, it can be shown that for a coax of inner and outer radii a and b,

$$\mathcal{R} = \sqrt{\frac{f\mu}{\pi\sigma}}\left(\frac{1}{a} + \frac{1}{b}\right),$$

while for a parallel-plate transmission line of width W,

$$\mathcal{R} = \frac{4\pi}{W}\sqrt{\frac{f\mu}{\pi\sigma}},$$

in terms of conductivity σ and permeability μ of the T.L. conductors.

1. lossless line solutions can be readily modified to account for loss effects introduced by Ohmic energy losses in $\mathcal{R}$ and $\mathcal{G}$,
2. lossless line results we have learned up till now are by and large valid even on lossy lines provided that
   (a) frequency ω is sufficiently large, and
   (b) voltage and current solutions $V^{\pm}e^{\pm j\beta d}$ and $\frac{V^{\pm}}{\pm Z_o}e^{\pm j\beta d}$ are modified by multiplying an attenuation term $e^{\pm\alpha d}$ which only matters in practice when $d \gg \lambda$.

- Note that lossless line solutions of telegrapher's equations can be restated as

$$V = V^{\pm}e^{\pm\gamma d} \text{ and } I = \frac{V^{\pm}}{\pm Z_o}e^{\pm\gamma d},$$

where

$$\gamma = j\beta = j\omega\sqrt{\mathcal{LC}} = \sqrt{(j\omega\mathcal{L})(j\omega\mathcal{C})} \text{ and } Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \sqrt{\frac{j\omega\mathcal{L}}{j\omega\mathcal{C}}}.$$

  – Replacing $j\omega\mathcal{L}$ by $j\omega\mathcal{L} + \mathcal{R}$, and $j\omega\mathcal{C}$ by $j\omega\mathcal{C} + \mathcal{G}$, we obtain

$$\gamma = \sqrt{(j\omega\mathcal{L} + \mathcal{R})(j\omega\mathcal{C} + \mathcal{G})} \text{ and } Z_o = \sqrt{\frac{j\omega\mathcal{L} + \mathcal{R}}{j\omega\mathcal{C} + \mathcal{G}}}$$

  in the lossy case.

While waves governed by lossy γ and Z_o (see margin) can exhibit substantially different behavior than the lossless waves (examined in the previous sections), at high frequencies the wave properties are reasonably similar as alluded in item (2) above. We next examine this simplified high-frequency limit.

$$\gamma = \sqrt{(j\omega\mathcal{L}+\mathcal{R})(j\omega\mathcal{C}+\mathcal{G})}$$

$$Z_o = \sqrt{\frac{j\omega\mathcal{L}+\mathcal{R}}{j\omega\mathcal{C}+\mathcal{G}}}$$

- At high frequencies ω, such that $\omega\mathcal{L} \gg \mathcal{R}$ and $\omega\mathcal{C} \gg \mathcal{G}$, we have
  - characteristic impedance

    $$Z_o = \sqrt{\frac{j\omega\mathcal{L}+\mathcal{R}}{j\omega\mathcal{C}+\mathcal{G}}} \approx \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}$$

    just as in the lossless case[1], and
  - complex propagation constant

    $$\begin{aligned}\gamma &= \sqrt{(j\omega\mathcal{L}+\mathcal{R})(j\omega\mathcal{C}+\mathcal{G})} = j\omega\sqrt{\mathcal{LC}}\sqrt{1+\frac{\mathcal{R}}{j\omega\mathcal{L}}}\sqrt{1+\frac{\mathcal{G}}{j\omega\mathcal{C}}} \\ &\approx j\omega\sqrt{\mathcal{LC}}(1+\frac{\mathcal{R}}{j2\omega\mathcal{L}})(1+\frac{\mathcal{G}}{j2\omega\mathcal{C}}) \approx j\omega\sqrt{\mathcal{LC}}+\frac{1}{2}(\frac{\mathcal{R}}{Z_o}+\mathcal{G}Z_o) \\ &= j\beta+\alpha\end{aligned}$$

    with

    $$\beta \approx \omega\sqrt{\mathcal{LC}} \quad \text{and} \quad \alpha \approx \beta(\frac{\mathcal{R}}{2\omega\mathcal{L}}+\frac{\mathcal{G}}{2\omega\mathcal{C}}) = \frac{1}{2}(\frac{\mathcal{R}}{Z_o}+\mathcal{G}Z_o).$$

[1] In fact $Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}$ is *exact* even for a lossy line if $\frac{\mathcal{L}}{\mathcal{R}} = \frac{\mathcal{C}}{\mathcal{G}}$.

- Note that $\beta = \frac{2\pi}{\lambda}$ is the same as in the lossless case, and since

$$\alpha \approx \beta(\frac{\mathcal{R}}{2\omega\mathcal{L}} + \frac{\mathcal{G}}{2\omega\mathcal{C}}) \ll \beta,$$

  the "penetration depth" $\delta \equiv \frac{1}{\alpha}$ of voltage and current waves on the TL is much longer than a wavelength $\lambda = \frac{2\pi}{\beta}$ in this regime.

In summary, in the high-frequency regime, characteristic impedance Z_o and wavenumber β are (practically) the same as they are on lossless lines, but signals do attenuate by a factor $e^{\pm\alpha d}$ which should not be (and cannot be) neglected over long distances d exceeding many wavelengths λ.

- At lower frequencies where the above approximations cannot be justified, a more careful analysis of lossy line equations is warranted.

- Finally, for an *air-filled coax* with inner and outer radii a and b, it can be shown that the attenuation constant

$$\alpha = \frac{1}{2}\frac{\mathcal{R}}{Z_o} = \frac{1}{2}\frac{\sqrt{\frac{f\mu_o}{\pi\sigma}}\frac{1}{b}(1+\frac{b}{a})}{\frac{\eta_o}{2\pi}\ln(\frac{b}{a})},$$

  which minimizes, at a fixed outer radius b, for $\frac{b}{a} \approx 3.6$, which in turn results in an "optimal" characteristic impedance of

$$Z_o = \frac{\eta_o}{2\pi}\ln(\frac{b}{a}) = 60\ln(\frac{b}{a})\,\Omega \approx 75\,\Omega$$

  for the same coax. Note that this result is independent of σ, the conductivity of inner and outer conductors of the coax.

- For a *dielectric filled coax* having $\epsilon = \frac{9}{4}\epsilon_o$ — implying $v_p = \frac{2}{3}c = 2\times 10^8$ m/s — the same ratio $\frac{b}{a} \approx 3.6$ of outer and inner conductor radii leads to $Z_o \approx 50\,\Omega$, the most common Z_o encountered in practical applications.
- The above result should also explain why having a thicker coax — larger b — is better when losses are a concern.