36 Smith Chart and VSWR

- Consider the general phasor expressions

$$V(d) = V^+ e^{j\beta d}(1 + \Gamma_L e^{-j2\beta d}) \text{ and } I(d) = \frac{V^+ e^{j\beta d}(1 - \Gamma_L e^{-j2\beta d})}{Z_o}$$

describing the voltage and current variations on TL's in sinusoidal steady-state.

  - Unless $\Gamma_L = 0$, these phasors contain reflected components, which means that voltage and current variations on the line "contain" standing waves.

In that case the phasors go through cycles of magnitude variations as a function of d, and in the voltage magnitude in particular (see margin) varying as

$$|V(d)| = |V^+||1 + \Gamma_L e^{-j2\beta d}| = |V^+||1 + \Gamma(d)|$$

takes maximum and minimum values of

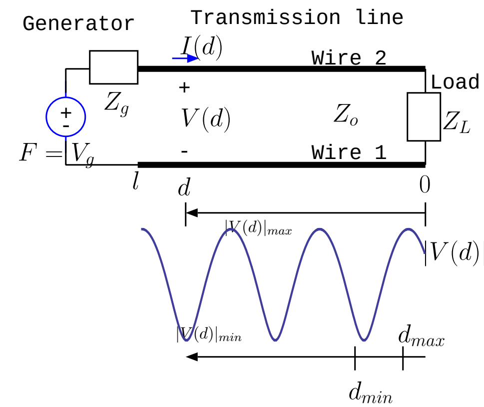
$$|V(d)|_{max} = |V^+|(1 + |\Gamma_L|) \text{ and } |V(d)|_{min} = |V^+|(1 - |\Gamma_L|)$$

at locations $d = d_{max}$ and d_{min} such that

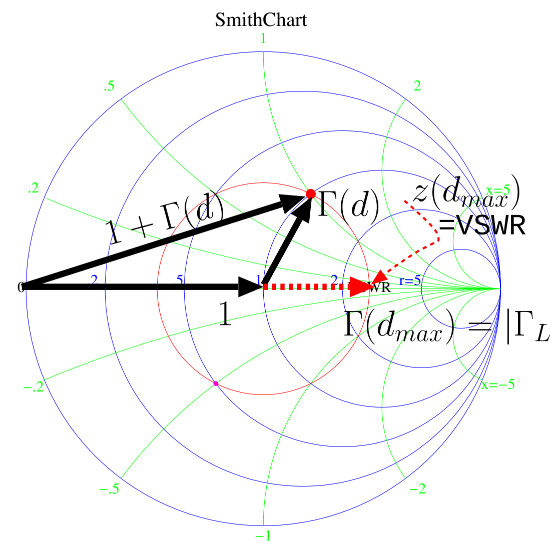
$$\Gamma(d_{max}) = \Gamma_L e^{-j2\beta d_{max}} = |\Gamma_L| \text{ and } \Gamma(d_{min}) = \Gamma_L e^{-j2\beta d_{min}} = -|\Gamma_L|,$$

and

$$d_{max} - d_{min} \text{ is an odd multiple of } \frac{\lambda}{4}.$$



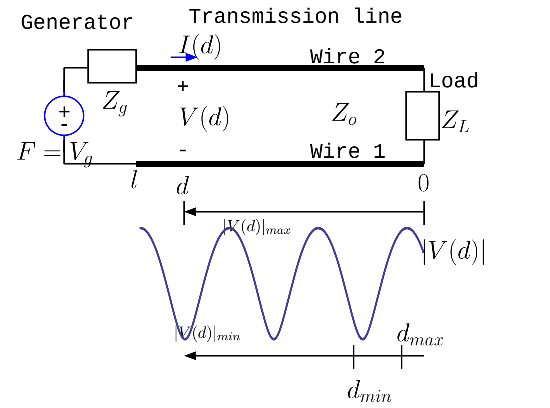
Complex addition displayed graphically superposed on a Smith Chart



$|1 + \Gamma(d)|$ maximizes for $d = d_{max}$

$|1 + \Gamma(d)|$ minimizes for $d = d_{min}$ such that $\Gamma(d_{min}) = -\Gamma(d_{max})$

- These results can be most easily understood and verified graphically on a SC as shown in the margin.



- We define a parameter known as **voltage standing wave ratio**, or **VSWR** for short, by

$$\text{VSWR} \equiv \frac{|V(d_{max})|}{|V(d_{min})|} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \quad \Leftrightarrow \quad |\Gamma_L| = \frac{\text{VSWR}-1}{\text{VSWR}+1}.$$

Notice that the VSWR and $|\Gamma_L|$ form a **bilinear transform pair** just like

$$z = \frac{1+\Gamma}{1-\Gamma} \quad \Leftrightarrow \quad \Gamma = \frac{z-1}{z+1}.$$

Since

$$\Gamma(d_{max}) = |\Gamma_L| \quad \Rightarrow \quad \text{VSWR} = \frac{1+\Gamma(d_{max})}{1-\Gamma(d_{max})},$$

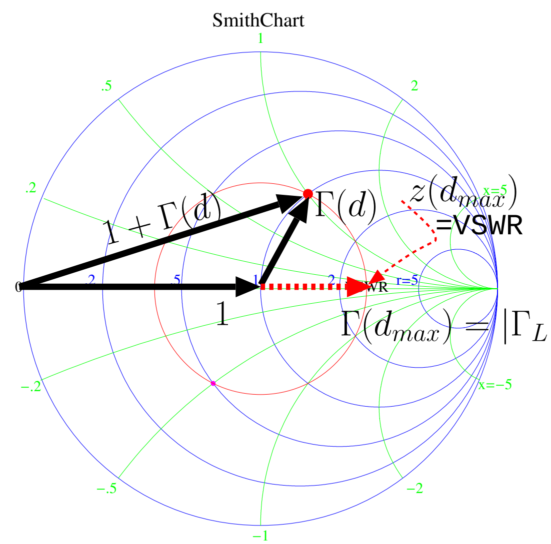
this analogy between the transform pairs also implies that

$$z(d_{max}) = \text{VSWR},$$

as explicitly marked on the the SC shown in the margin . Consequently,

- the VSWR of any TL can be directly read off from its SC plot as the normalized impedance value $z(d_{max})$ on constant-$|\Gamma_L|$ circle crossing the positive real axis of the complex plane.

Complex addition displayed graphically superposed on a Smith Chart

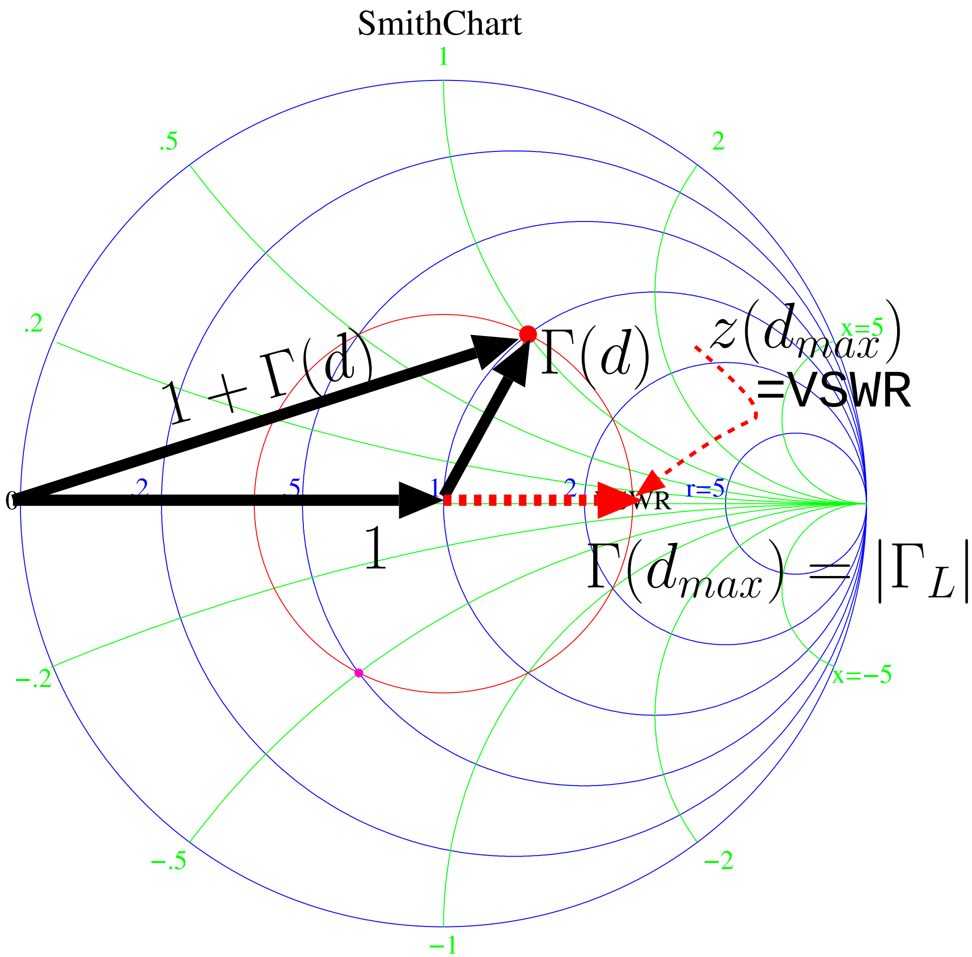


$|1+\Gamma(d)|$ maximizes for $d = d_{max}$

$|1+\Gamma(d)|$ minimizes for $d = d_{min}$ such that $\Gamma(d_{min}) = -\Gamma(d_{max})$

- The extreme values the VSWR can take are:

  1. VSWR=1 if $|\Gamma_L| = 0$ and the TL carries no reflected wave.
  2. VSWR=∞ if $|\Gamma_L| = 1$ corresponding to having a short, open, or a purely reactive load that causes a total reflection.



$|1 + \Gamma(d)|$ maximizes for $d = d_{max}$

$|1 + \Gamma(d)|$ minimizes for $d = d_{min}$ such that $\Gamma(d_{min}) = -\Gamma(d_{max})$

- In the lab it is easy and useful to determine the VSWR and d_{max} or d_{min} of a TL circuit with an unknown load, since

1. given the VSWR,

$$|\Gamma_L| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1}$$

   is easily determined, and

2. given d_{max} or d_{min} the complex Γ_L or its transform z_L can be easily obtained.

Say d_{max} is known: then,

- since (as we have seen above)

$$\Gamma(d_{max}) = \Gamma_L e^{-j2\beta d_{max}} = |\Gamma_L|$$

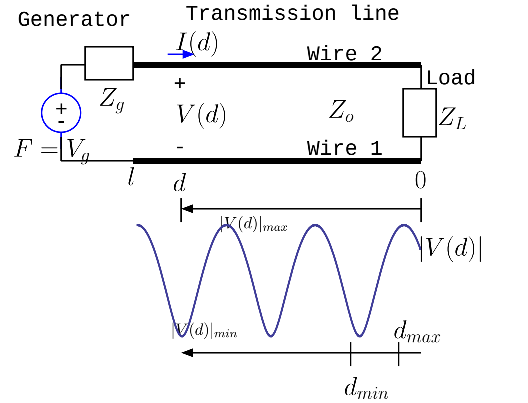
  it follows that

$$\Gamma_L = |\Gamma_L| e^{j2\beta d_{max}} \quad \Rightarrow \quad z_L = \frac{1 + \Gamma_L}{1 - \Gamma_L}.$$

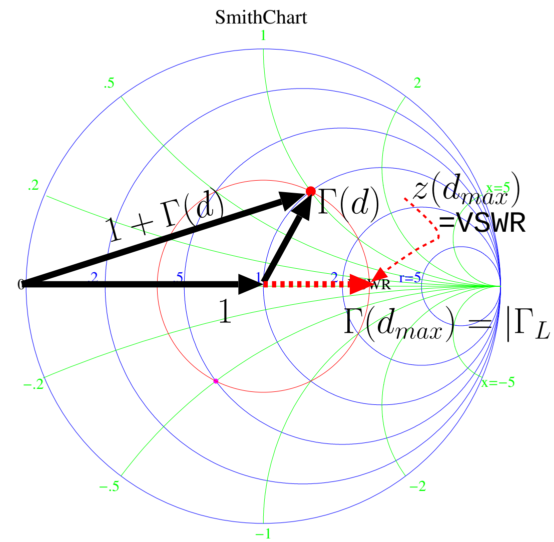
- alternatively, z_L can be obtained directly on the SC by rotating counter-clockwise by d_{max} from the location of

$$z(d_{max}) = \text{VSWR}.$$

These techniques are illustrated in the next example.



Complex addition displayed graphically superposed on a Smith Chart



$|1 + \Gamma(d)|$ maximizes for $d = d_{max}$

$|1 + \Gamma(d)|$ minimizes for $d = d_{min}$ such that $\Gamma(d_{min}) = -\Gamma(d_{max})$

Example 1: An unknown load Z_L on a $Z_o = 50\,\Omega$ TL has

$$V(d_{min}) = 20\,\text{V}, \quad , d_{min} = 0.125\lambda \;\text{ and }\; \text{VSWR}{=}4.$$

Determine (a) the load impedance Z_L, and (b) the average power P_L absorbed by the load.

Solution: (a) As shown in the top SC in the margin, VSWR=4 is entered in the SC as $z(d_{max}) = 4 + j0$, and constant $|\Gamma_L|$ circle is then drawn (red circle) passing through $z(d_{max}) = 4$.

Right across $z(d_{max}) = 4$ on the circle is $z(d_{min}) = 0.25$.

A counter-clockwise rotation from $z(d_{min}) = 0.25$ by one fourth of a full circle corresponding to a displacement of $d_{min} = 0.125\lambda$ (a full circle corresponds to a $\lambda/2$ displacement) takes us to

$$z_L \approx 0.4706 - j0.8823$$

as shown in the second SC. Hence, this gives

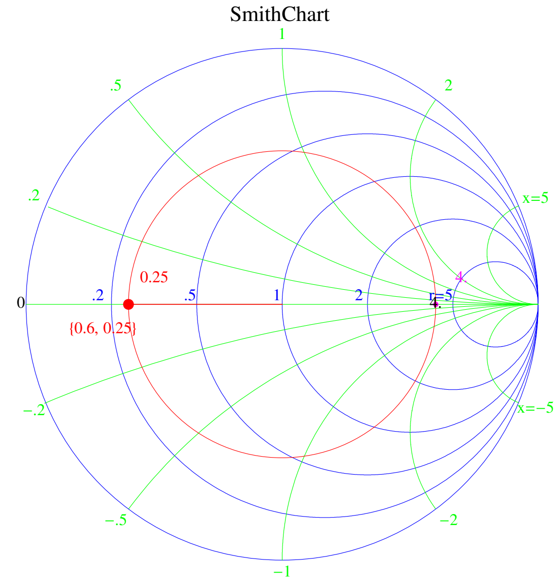
$$Z_L = Z_o z_L = 50(0.4706 - j0.8823) = 23.53 - j44.12\,\Omega.$$

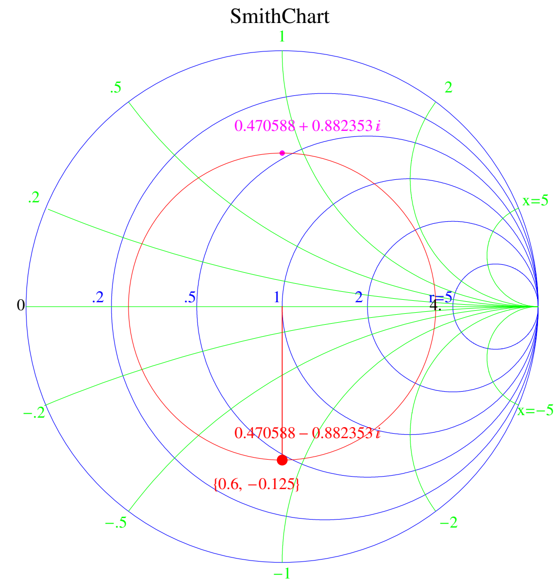
(b) We will calculate P_L by using $V(d_{min})$ and $I(d_{min})$. Since

$$z(d_{min}) = 0.25 \;\text{ it follows that }\; Z(d_{min}) = \frac{1}{4} 50\,\Omega = 12.5\,\Omega.$$

Therefore the voltage and current phasors at the voltage minimum location are

$$V(d_{min}) = 20\,\text{V} \;\text{ and }\; I(d_{min}) = \frac{20\,\text{V}}{12.5\,\Omega}.$$



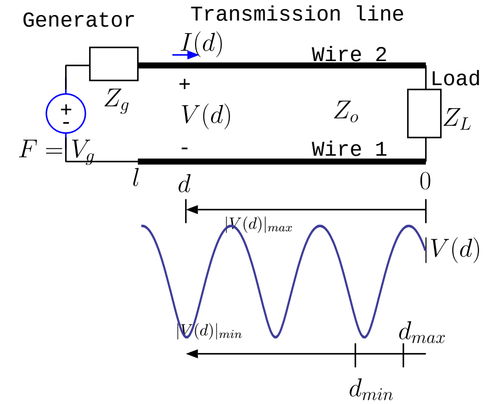


Average power transported toward the load at $d - d_{min}$ is, therefore,

$$P(d_{min}) = \frac{1}{2}\text{Re}\{V(d_{min})I(d_{min})^*\} = \frac{1}{2}\text{Re}\{20\frac{20}{12.5}\} = \frac{400}{25}\,\text{W} = 16\,\text{W}.$$

Since the TL is assumed to be lossless we should have

$$P_L = P(d_{min}) = 16\,\text{W}.$$



Example 2: If the TL circuit in Example 1 has $l = 0.625\lambda$, and a generator with an internal impedance $Z_g = 50\,\Omega$, determine the generator voltage V_g.

Solution: Given that $l = 0.625\lambda$ and $d_{min} = 0.125\lambda$, we note that there is just one half-wave transformer between $l = 0.625\lambda$ and $d_{min} = 0.125\lambda$. Therefore

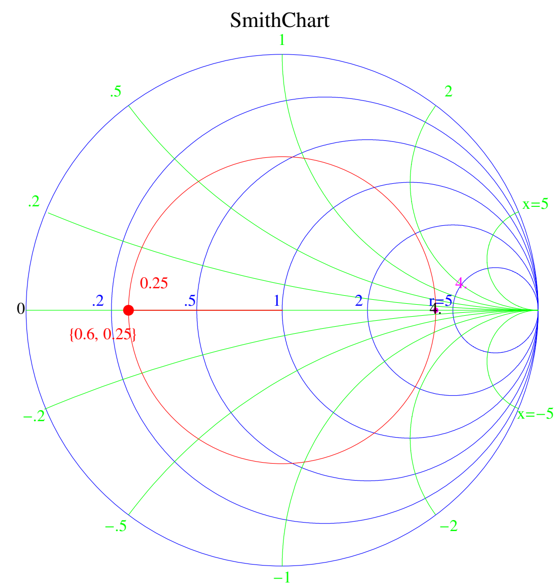
$$V_{in} = -V(d_{min}) = -20\,\text{V} \quad \text{and} \quad Z_{in} = Z(d_{min}) = 12.5\,\Omega.$$

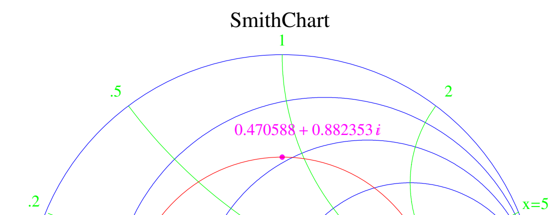
But also

$$V_{in} = V_g \frac{Z_{in}}{Z_g + Z_{in}}.$$

Consequently,

$$V_g = V_{in}\frac{Z_g + Z_{in}}{Z_{in}} = -20\frac{50 + 12.5}{12.5} = -20\frac{62.5}{12.5} = -100\,\text{V}.$$





Example 3: Determine V^+ and V^- in the circuit of Examples 1 and 2 above such that the voltage phasor on the line is given by

$$V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}.$$

Solution: Looking back to Example 1 (also see the SC's in the margin), we first note that

$$|\Gamma_L| = \frac{\mathrm{VSWR}-1}{\mathrm{VSWR}+1} = \frac{4-1}{4+1} = 0.6 = \Gamma(d_{max}) = -\Gamma(d_{min}).$$

Hence, evaluating $V(d)$ at $d = d_{min}$, we have

$$\begin{aligned} V(d_{min}) &= V^+ e^{j\beta d_{min}}(1+\Gamma(d_{min})) \\ &= V^+(e^{j\frac{2\pi}{\lambda}\frac{\lambda}{8}})(1+(-0.6)) = 0.4e^{j\frac{\pi}{4}}V^+ = 20\,\mathrm{V}, \end{aligned}$$

from which

$$V^+ = 50e^{-j\frac{\pi}{4}}\,\mathrm{V}.$$

Since

$$\Gamma_L = \Gamma(0) = \Gamma(d_{min})e^{j2\beta d_{min}} = -0.6e^{j\frac{\pi}{2}},$$

it follows that

$$V^- = \Gamma_L V^+ = -0.6e^{j\frac{\pi}{2}} \times 50e^{-j\frac{\pi}{4}} = -30e^{j\frac{\pi}{4}}\,\mathrm{V}.$$

Example 4: Determine the load voltage and current $V_L = V(0)$ and $I_L = I(0)$ in the circuit of Examples 1-3 above.

Solution: In general,

$$V(d) = V^+ e^{j\beta d} - V^- e^{-j\beta d} \text{ and } I(d) = \frac{V^+ e^{j\beta d} - V^- e^{-j\beta d}}{Z_o}.$$

Therefore,

$$V_L = V(0) = V^+ + V^- \text{ and } I_L = I(0) = \frac{V^+ - V^-}{Z_o}.$$

Using $Z_o = 50\,\Omega$ and

$$V^+ = 50e^{-j\frac{\pi}{4}}\text{ V} \text{ and } V^- = -30e^{j\frac{\pi}{4}}\text{ V}$$

from Example 3, we find that

$$V_L = 50e^{-j\frac{\pi}{4}} - 30e^{j\frac{\pi}{4}}\text{ V} \text{ and } I_L = \frac{50e^{-j\frac{\pi}{4}} + 30e^{j\frac{\pi}{4}}}{50} = e^{-j\frac{\pi}{4}} + 0.6e^{j\frac{\pi}{4}}\text{ A}.$$