29 Bounce diagrams

- Last lecture we obtained the **impulse-response functions**

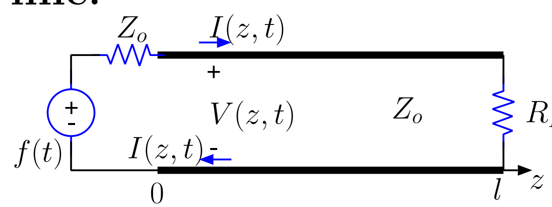
$$V(z,t) = \tau_g[\delta(t - \frac{z}{v}) + \Gamma_L \delta(t + \frac{z}{v} - \frac{2l}{v})]$$

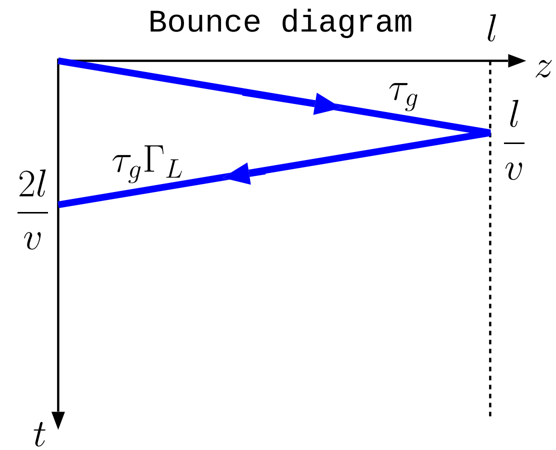
and

$$I(z,t) = \frac{\tau_g}{Z_o}[\delta(t - \frac{z}{v}) - \Gamma_L \delta(t + \frac{z}{v} - \frac{2l}{v})]$$

for the voltage and current in the TL circuit shown in the margin where the source is matched to the line so that $\tau_g = \frac{1}{2}$ — circuit response with an arbitrary input $f(t)$ is obtained by convolving these with $f(t)$ (as shown in Example 1 in last lecture).

Source matched to line:

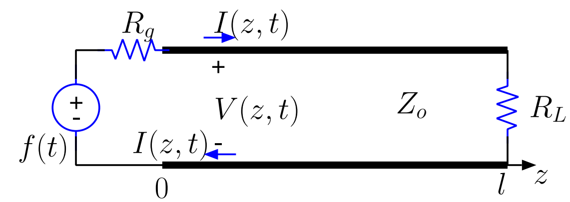


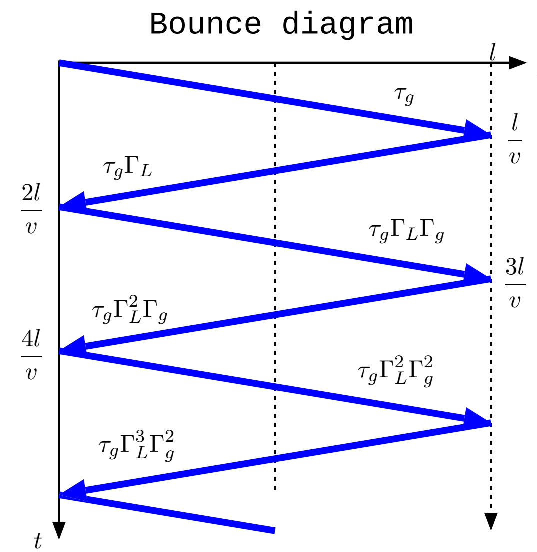


- The impulse-response for $V(z,t)$ is depicted in the margin in the form of a **bounce diagram**, in which
  - the trajectories of the impulses constituting the impulse response are plotted, with
    - z axis in the horizontal, and
    - t axis in the vertical extending from top to bottom
  - and coefficients of each impulse noted in the diagram next to the trajectory lines.
  - the blue line sloping down on the top is a depiction of forward propagating impulse $\tau_g \delta(t - \frac{z}{v})$,

– the next line down is the depiction of backward propagating impulse $\tau_g \Gamma_L \delta(t + \frac{z}{v} - \frac{2l}{v})$.

Bounce diagrams are graphical representations of impulse response functions derived in TL circuit problems, and are *primarily used to determine the impulse response functions*, rather than the other way around as will be illustrated below.





- We show in the margin a circuit with an arbitrary

$$R_g \quad \text{and} \quad \tau_g = \frac{Z_o}{R_g + Z_o},$$

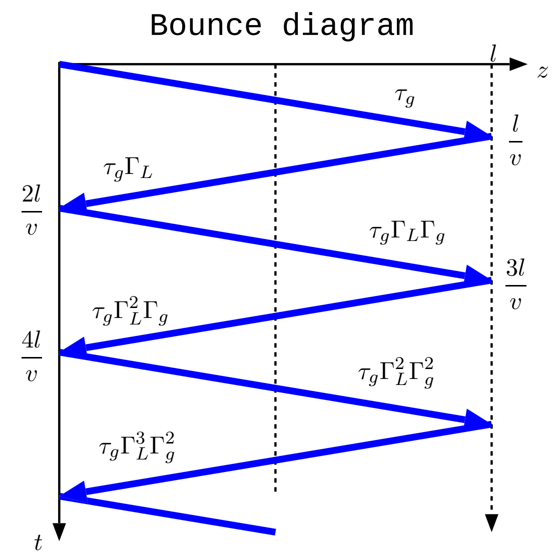
for which the bounce diagram is not terminated at $t = \frac{2l}{v}$ because the backward propagating impulse on the line arriving at $z = 0$ at time $t = \frac{2l}{v}$ is reflected from $z = 0$ with a reflection coefficient of

$$\Gamma_g = \frac{R_g - Z_o}{R_g + Z_o}.$$

  – Reflections of negative-going impulses incident on the source circuit are justified because these impulses just see the resistor R_g at the generator end — the source voltage $f(t) = \delta(t)$ is by then just a short is series with R_g — unmatched to Z_o, just like the forward going impulses seeing a load R_L unmatched to Z_o and reflecting with a coefficient

$$\Gamma_L = \frac{R_L - Z_o}{R_L + Z_o}.$$

- Once the bounce diagram for voltage has been constructed as shown above, then the impulse response can be written by inspection as



$$\begin{aligned} V(z,t) &= \tau_g \sum_{n=0}^{\infty} (\Gamma_L\Gamma_g)^n \delta(t - \frac{z}{v} - n\frac{2l}{v}) \\ &+ \tau_g\Gamma_L \sum_{n=0}^{\infty} (\Gamma_L\Gamma_g)^n \delta(t + \frac{z}{v} - (n+1)\frac{2l}{v}). \end{aligned}$$

Also,

$$\begin{aligned} I(z,t) &= \frac{\tau_g}{Z_o} \sum_{n=0}^{\infty} (\Gamma_L\Gamma_g)^n \delta(t - \frac{z}{v} - n\frac{2l}{v}) \\ &- \frac{\tau_g}{Z_o}\Gamma_L \sum_{n=0}^{\infty} (\Gamma_L\Gamma_g)^n \delta(t + \frac{z}{v} - (n+1)\frac{2l}{v}). \end{aligned}$$

  - It can also be shown that the first term of $V(z,t)$ above is derived from the formal solution of the equation

$$V^+(t) = \tau_g \delta(t) + \Gamma_L\Gamma_g V^+(t - \frac{2l}{v})$$

which is obtained from

$$I(0,t) = \frac{\delta(t) - V(0,t)}{R_g}$$

enforced at $z = 0$. We have effectively by-passed such a formal approach to the problem by using the bounce diagram technique.

- These awful series formulae above are hardly needed in most applications when only the first few terms of the series are sufficient for reasonably accurate results (like in the next example).

Example 1: Consider a TL circuit where $Z_o = 50\,\Omega$, $v = c$, $l = 2400$ m, $R_g = 0$, and $R_L = 100\,\Omega$. Determine and plot $V(1200, t)$ if $f(t) = u(t)$.

Solution: For this circuit

$$\tau_g = \frac{Z_o}{R_g + Z_o} = 1, \quad \Gamma_g = \frac{R_g - Z_o}{R_g + Z_o} = -1, \quad \text{and} \quad \Gamma_L = \frac{R_L - Z_o}{R_L + Z_o} = \frac{1}{3}.$$

Also, the transit time across the TL is

$$\frac{l}{v} = \frac{2400\,\text{m}}{300 \times 10^6\,\text{m/s}} = 8\,\mu\text{s}.$$

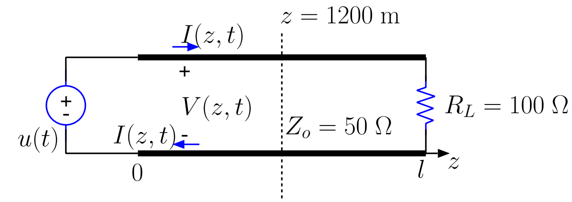
From the bounce diagram shown in the margin, the impulse response for $z = 1200$ m (the location marked by the vertical dashed line) is found to be

$$V(1200, t) = \delta(t-4) + \frac{1}{3}\delta(t-12) - \frac{1}{3}\delta(t-20) - \frac{1}{9}\delta(t-28) + \frac{1}{9}\delta(t-36) + \cdots$$

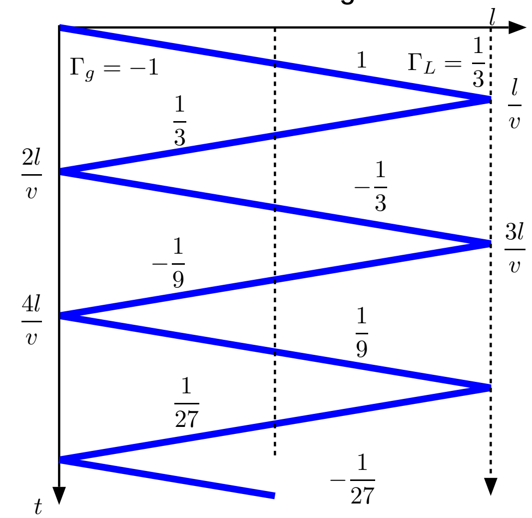
Replacing the $\delta(t)$ in this expression with the unit-step $u(t)$, the specified source function $f(t)$, we get

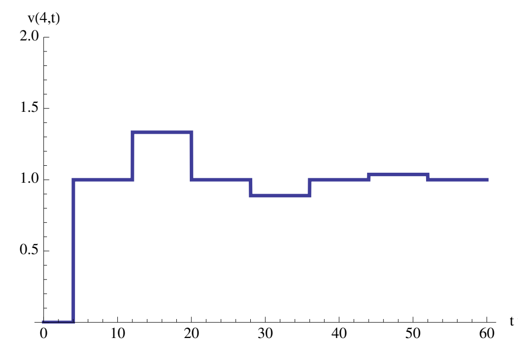
$$V(1200, t) = u(t-4) + \frac{1}{3}u(t-12) - \frac{1}{3}u(t-20) - \frac{1}{9}u(t-28) + \frac{1}{9}u(t-36) + \cdots$$

which is plotted in the margin.



Bounce diagram



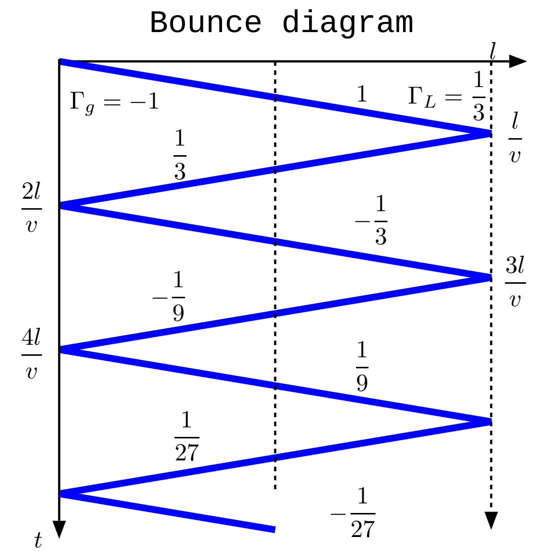


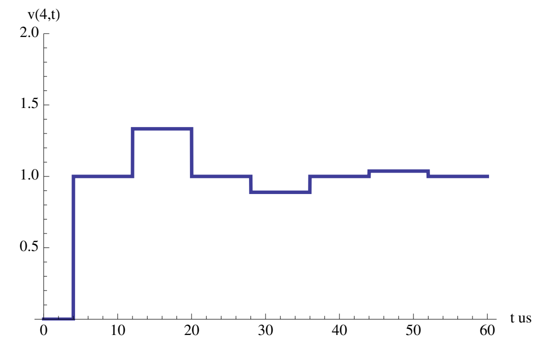
Animated version of this is linked in the class calendar.

- Note that as $t \to \infty$, $V(1200, t) \to 1$ V in Example 1, as if DC conditions prevail and the TL becomes a pair of wires in the lumped circuit sense.

  – DC steady-state corresponds to $\omega = 0$ and signal wavelength $\lambda \to \infty$. In that limit $l \ll \lambda$ is always valid and TL can be treated like an ordinary lumped circuit.

  – Of course this simplification can only occur with $f(t) \propto u(t)$, or its delayed versions, which are all asymptotically DC in $t \to \infty$ limit. The simplification does not apply for $f(t) = \sin(\omega t)u(t)$, for example.





Example 2: In the TL circuit described in Example 1, determine $V(z,t)$ and $I(z,t)$ for a new source signal $f(t) = \text{rect}(\frac{t}{T}) + 2\text{rect}(\frac{t-T}{T})$, $T = 1\ \mu$s. Plot $V(z,t)$ versus z at $t = 3\ \mu s$ and $t = 11\ \mu s$.

Solution: With $\tau_g = 1$, $\Gamma_g = -1$, $\Gamma_L = \frac{1}{3}$, and $\frac{2l}{c} = 16\,\mu$s, we obtain, by convolving with the general impulse response, the voltage response

$$V(z,t) = \sum_{n=0}^{\infty} (-\frac{1}{3})^n f(t - \frac{z}{c} - n16) + \frac{1}{3} \sum_{n=0}^{\infty} (-\frac{1}{3})^n f(t + \frac{z}{c} - (n+1)16)$$

where $\frac{z}{c}$ is to be entered in μs units. Also,

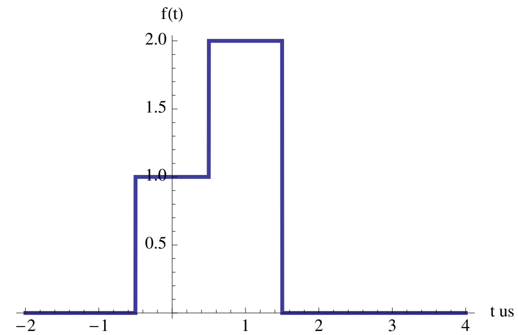
$$I(z,t) = \frac{1}{50} \sum_{n=0}^{\infty} (-\frac{1}{3})^n f(t - \frac{z}{c} - n16) - \frac{1}{50}\frac{1}{3} \sum_{n=0}^{\infty} (-\frac{1}{3})^n f(t + \frac{z}{c} - (n+1)16).$$

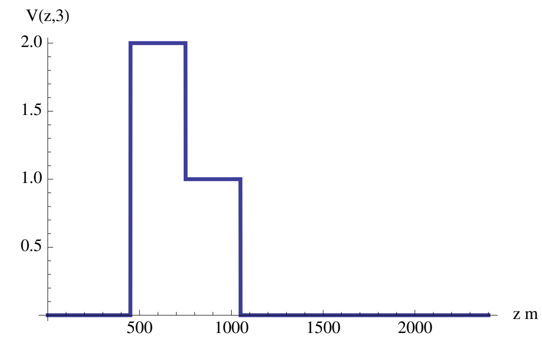
At $t = 3\ \mu s$, the voltage variation is

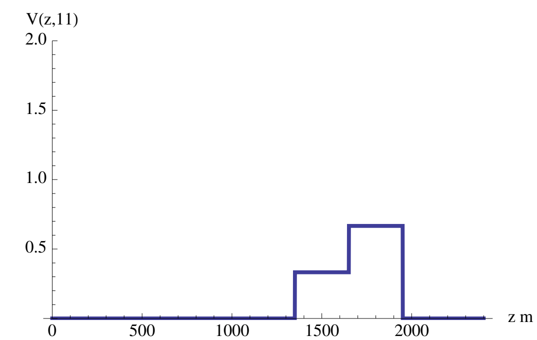
$$V(z,3) = \sum_{n=0}^{\infty} (-\frac{1}{3})^n f(3 - \frac{z}{c} - n16) + \frac{1}{3} \sum_{n=0}^{\infty} (-\frac{1}{3})^n f(3 + \frac{z}{c} - (n+1)16),$$

which is plotted in the margin using $f(t) = \text{rect}(t) + 2\text{rect}(t-1)$. Likewise, at $t = 11\ \mu s$,

$$V(z,11) = \sum_{n=0}^{\infty} (-\frac{1}{3})^n f(11 - \frac{z}{c} - n16) + \frac{1}{3} \sum_{n=0}^{\infty} (-\frac{1}{3})^n f(11 + \frac{z}{c} - (n+1)16).$$







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